CS200 Worksheet - Counting and Probability

- 1. How many DNA sequences of length 6 $({G, A, C, T}^6)$ (you do not need to simplify your answer)
 - (a) Do not contain T?
 - (b) Contain the base A?
 - (c) Do not contain all 4 base pairs?
 - (d) Contain all 4 base pairs?
 - (e) Contain the ordered sequence CAT? (Careful of overcounting.)
 - (f) Contain exactly 2 of the 4 base pairs?
- 2. Explain why there are at least six people in California (population: 37 million) with the same three initials who were born on the same day of the year (but not necessarily in the same year). Assume that everyone has three initials and there are 366 days in the year, including leap years. (Remember the pigeon hole principle!)
- 3. A coin is flipped 10 times. If the order of the outcomes matters, what is the probability that
 - (a) The sequence alternates heads and tails?
 - (b) There are exactly 5 heads and 5 tails?
 - (c) There are at most 3 tails?
- 4. In roulette, you spin a ball on a wheel to get one of 38 possible numbers. There are 18 black numbers, 18 red numbers, and 2 white numbers.
 - (a) What is the probability that you get a red number?
 - (b) What is the probability that you get a black number twice in a row? What is the probability that you get a white number at any point in 5 spins.
- 5. Suppose you have a loaded di, where a 3 is twice as likely to appear as any of the other five outcomes.
 - (a) What is the probability of each outcome?
 - (b) What is the probability of getting an even outcome?