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Goals

- Prove whether relation is equivalence relation
- Use summation notation for time complexity

Announcements

- Reflection: Practical, walk through more examples b/f group work, go over HW, go over plicker, better textbooks, group work
- Wed reminder

Equivalence Class $\rightsquigarrow$ Equivalence Relation

Given a set A, can divide into disjoint subsets that have common property.

disjoint: no overlap / intersection $= \emptyset$

ex: $A = \{-1, 1, 2, 3\}$

Same Sign

$\{-1\}$ $\{1, 2, 3\}$

Even vs. Odd

$\{-1, 1, 3\}$ $\{2\}$

Equal to Self

$\{-1\}$ $\{1\}$ $\{2\}$ $\{3\}$

Every element in subset is equivalent to every other element in subset.

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Another way to express this equivalence is through equivalence relation:

$(a,b) \in R$, R an equivalence relation, iff a,b in same equivalence class

<u>def</u>: $R \subseteq A \times A$, R reflexive, symmetric, transitive

$\forall a \in A, (a,a) \in R$

$\forall a,b \in A, (a,b) \in R \rightarrow (b,a) \in R$

$\forall a,b,c \in A, (a,b) \wedge (b,c) \rightarrow (a,c)$

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A) Equivalence Relation

B) Not reflexive

C) Not symmetric

D) Not transitive

1. $R = \{(a,b) \in \mathbb{R} \times \mathbb{R} : a-b \in \mathbb{Z}\}$

A) <u>Equivalence Relation</u>

1. Let $a \in \mathbb{R}$. Then $a-a=0$, and $0 \in \mathbb{Z}$, So Reflexive condition satisfied.

2. Let $a,b \in \mathbb{R}$. For the forward direction, Assume $(a-b) \in \mathbb{Z}$. Then $-(a-b) \in \mathbb{Z}$.
For the backward direction

3. $a-b = x \in \mathbb{Z}$ $\quad b-c = y \in \mathbb{Z}$

$x+y \in \mathbb{Z}$

$x+y = (a-b)+(b-c) = a-c \Rightarrow a-c \in \mathbb{Z}$ ✓ Transitive

2. $R \subseteq \mathbb{Z} \times \mathbb{Z}$, $(a,b) \in R \Leftrightarrow a|b$

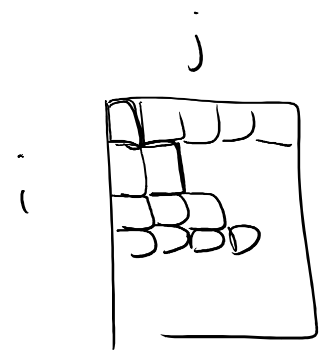
C) NOT Symmetric

$a|b \not\Leftrightarrow b|a$ $\quad$ 2 divides 4 but 4 does not divide 2

Input: Adj Matrix A for $G=(V,E)$ (undirected, unweighted, no self loops)

Output:

```
S=0
for i=1 to |V|:
    for j=1 to i:
        S += A[i,j]
return S
```



How many operations?

For loop → summation. Write outer to inner

$$\sum_{i=1}^{|V|}\left(\sum_{j=1}^{i} 1\right) = \sum_{i=1}^{|V|} i$$

outer loop

inner loop

# of opera

Analyze inner to outer

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$$\sum_{i=1}^{|V|} i = 1 + 2 + 3 + 4 + \cdots + |V|-1 + |V|$$

$|V|+1$ $|V|+1$ $|V|+1$

How many pairs? $\frac{|V|}{2}$

Total: $\frac{|V|}{2}(|V|+1) = \frac{|V|^2}{2} + |V| = O(|V|^2)$

\# pairs amount per pair

$$\sum_{i=1}^{n} i = \frac{n}{2}(n+1)$$

$$\sum_{i=a}^{b} i = \frac{(a-b+1)}{2}(a+b)$$

So from previous, # ops $= O(|V|^2)$

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Input: Adj Matrix A for $G=(V,E)$ (undirected, unweighted, no self loops)

Output:

1. $S=0$
2. for $i=1$ to $|V|$: } $O(|V|)$
3.   for $j=1$ to i: } $O(|V|)$
4. $S \mathrel{+}= A[i,j]$
5. return S

} $O(|V|^2)$

big-O is upper bound