CS200 - Problem Set 5

Due: Monday, March. 19 to Canvas before class

1. **[11 points]** Prove Algorithm 1 for binary search is correct using strong induction on n, where $n = f - s$. You should also use proof by cases for the if-elses. **Note:** This is a complex proof, and you will probably not get all of the parts correct. Just try your best :) Recall, if A is sorted in increasing order and no integers are repeated, that means $i < j$ if and only if $A[i] < A[j]$.

   Input : (1) Array A containing integers, where there are no repeated integers and the integers are sorted from smallest to largest, (2) an element V in A, and (3) two indeces s and f, where $s \leq f$ and the index of V is between s and f (inclusive)

   Output: Index j such that $A[j] = V$, and $s \leq j \leq f$.

   ```
   // Base Case
   if f − s = 0 then
   |   return s;
   end
   // Recursive step
   mid = ⌊(f + s)/2⌋;
   // ⌊·⌋ means round down to the nearest integer
   if A[mid] = V then
   |   return mid;
   else
   |   if A[mid] < V then
   |   |   return BinarySearch(A, V, mid + 1, f);
   |   else
   |   |   return BinarySearch(A, V, s, mid − 1);
   |   end
   end
   ```

   Algorithm 1: `BinarySearch`(A, V, s, f)

2. *Party-trick Proof* **[11 points]** Suppose you are at a party with 19 acquaintances (so there are 20 people at the party). Prove (using a proof by contradiction) that there must be at least two people at the party who talked to the same number of people over the course of the evening. (Note: we assume that if Alice talked to Bob, that also means that Bob talked to Alice.)

3. **[3 points]** Suppose you can prove a statement using induction. Can you also prove the same statement using strong induction? Explain.