

CS200 - Problem Set 3

Due: Monday, Mar. 5 to Canvas before class

Please read the sections of the syllabus on problem sets and honor code before starting this homework.

1. *Statements* [**2 point each**] Simplify each of the following expressions, where p denotes a statement, and T and F are the Boolean constants *true* and *false*. Hint: each answer is one of p , T , or F . No proof needed, no steps need be shown.

- (a) $T \wedge p$
- (b) $F \wedge p$
- (c) $T \vee p$
- (d) $F \vee p$
- (e) $p \vee p$
- (f) $p \wedge p$
- (g) $p \vee \neg p$

2. *Quantifiers*

Consider the following statement:

$$\forall x, \exists y : (y > x) \wedge (\forall z, ((z \neq y) \wedge (z > x)) \rightarrow (z > y))$$

- (a) [**3 points**] If the domain of x , y and z is \mathbb{Z} , state whether the statement is true or false. Justify your answer.
- (b) [**3 points**] If the domain of x , y and z is \mathbb{R} , state whether the statement is true or false. Justify your answer.

Hint: when you see $\forall x$, imagine that your enemy is trying to prove the statement false, and gets to choose any x they want, and you want to show that no matter what they do, the statement is still true. When you see $\exists y$ after $\forall x$, you get to pick y to try to show that you can counter your enemy's choice of x .

3. *Turning English Into Math* [**6 points each**]

When writing a proof, it is often helpful to use mathematical notation, rather than writing out the equivalent in English. This question will help you to practice this skill.

Let S be a set of students in a class and $f(s)$ be the score obtained by student s in an exam. Translate the English description of each predicate or statement below into a logical formula using quantifiers. When writing a formula for a statement or a predicate, you may use any propositions/predicates that you have previously defined. Use the \equiv symbol in your answer. For example, for part *a*, you should write $H(n) \equiv \dots$

To get full credit, your answer should only use mathematical notation, and your response should be approximately as concise as mine.

- (a) Predicate $H(n)$ asserts: n is the highest score that any student got on the exam.
- (b) Predicate $B(s)$ asserts: student s got the highest score.
- (c) Statement p asserts: at least two students got the highest score.
- (d) Predicate $M(n)$ asserts: if any two students got the same score, that score is at least n .
- (e) Predicate $R(s)$ asserts: student s got 10 points less than the highest score.
- (f) Statement t asserts: the second highest score in the class is 10 points less than the highest score.

4. *Implications* [3 points each]

There are many ways to represent a logical implication ($P \rightarrow Q$) in English. To make proofs more interesting to read, we often take advantage of these different ways of phrasing the same underlying mathematical statement. In the following, I will ask you to rewrite sentences in the form $p \rightarrow q$. For example, “I get a brain freeze if I eat ice cream” should be rewritten “I eat ice cream \rightarrow I get a brain freeze.” Try to reason these out based on the English meaning first. Try thinking about the four possible options (True/False for each of P and Q) and think about which make sense. If you are having trouble, check out p. 43 of Book of Proof, or problem 5 in Chapter 0 of DMOI (which has solutions).

- (a) I open my umbrella whenever it rains.
- (b) I miss class only if I am unwell.
- (c) You can't invent unless you are curious and knowledgeable.

5. *Logical Equivalences* [3 points each] The following are important logical equivalences. (They are worth memorizing!)

- (a) $p \rightarrow q \equiv \neg q \rightarrow \neg p$ (a statement is equivalent to its contrapositive)
- (b) $\neg(p \wedge q) \equiv \neg p \vee \neg q$ (DeMorgan's Law)
- (c) $\neg(p \vee q) \equiv \neg p \wedge \neg q$ (DeMorgan's Law)
- (d) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ (\wedge distributes over \vee)
- (e) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ (\vee distributes over \wedge)

Give an informal justification for each of (a), (b), and (c). That is, explain in a sentence or two why the equivalence is true. Show (d) through truth table. When writing the truth table, be sure to include columns for subexpressions, such as $p \wedge q$. The following is an example truth table for (e) showing in the last column the equivalence of output columns 2 and 5.

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$	$p \vee (q \wedge r) \leftrightarrow (p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	T	T	T	T
F	T	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F	T
F	F	T	F	F	F	T	F	T
F	F	F	F	F	F	F	F	T

6. **[11 points each]** If you did not get full credit for the induction question on the quiz, please re-solve. (The solutions to the quiz are under Files on Canvas; don't look until after the pset is due.) Additionally, solve question 19 in Chapter 10 of BOP. (The solutions to odd numbered problems are at the end of the text; don't look until after the pset is due.)
7. **[6 points]** **This problem is not due for this problem set, but will be part of the following problem set.**

What can you deduce from the following true statements? Please write the simplest new true statement possible. (Partial credit will be given for more complex statements.)

$$P \rightarrow R$$

$$Q \rightarrow R$$

$$P \vee Q$$

∴

(1)

8. **[6 points]** **This problem is not due for this problem set, but will be part of the following problem set.**

(Challenge problem) You meet two spiders on the road. Everyone knows that a spider either always tells the truth, or always lies. The first spider says, "If we are brothers, then we are both liars." The second spider says, "We are cousins or we are both liars." Are both spiders telling the truth? (Hint, create a truth table for their statements and consider the possible cases of each spider lying or telling the truth, and use deduction so see if there is a contradiction. Also, can brothers be cousins?)

9. How long did you spend on this homework?