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Quiz! (Work on: Prove sum of first n odd numbers is n^2)

Announcements:
- HW
    - Satisfactory
    - PDF
    - Test early
- Tutoring
    - Bonus

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Notice:

- Complete sentences or equations. Should be able to smoothly read aloud.

Q: Write inductive proof that $7^n - 1$ is a multiple of 6 for all integers $n \geq 0$.

(Hint: X is a multiple of 6 if $X = 6 \cdot m$ for some integer m.)

¶ Let $P(n)$ be the predicate $7^n - 1$ is a multiple of 6. We will prove via induction that $P(n)$ is true for all $n \geq 0$.

¶ Base Case: $P(0)$ is the statement $7^0 - 1$ is a multiple of 6.
$7^0 - 1 = 1 - 1 = 0 = 0 \cdot 6$, so the statement is true.

¶ Inductive Case: Let $k \geq 0$. Assume, for induction, that $P(k)$ is true. This means $7^k - 1 = 6 \cdot m$ for some integer m.
Multiplying both sides by 7, we get

$$7(7^k - 1) = 7^{k+1} - 7 = 7 \cdot 6m$$

Adding 6 to both sides, we get

$$7^{k+1} - 1 = 6(7m + 1).$$

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Since $7m-1$ is an integer, $7^{k+1}-1$ is divisible by 6. Thus $P(k+1)$ is true.

¶ Therefore, by induction on n, $P(n)$ is true for all $n \geq 0$.

Parts of Inductive Proof

1. Set-up (state problem, approach)
2. Base-case: (1st solution)
3. Inductive step/case: ($k^{th} \to (k+1)^{th}$ solution)
4. Conclusion (put a bow on it!)

Purpose

(Tell them what you're going to say, say it, tell them what you said)

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Proof Tips

- Don't try to figure out all steps before starting proof. The process of writing the proof will help you to figure it out.

- Phrase inductive assumption (P(k) is true) using as much math as possible

e.g. Instead of: $7^k - 1$ is divisible by 6,
Better: $7^k - 1 = 6m$ for an integer m

Prove: $2^n + 1 \leq 3^n$ for all integers $n \geq 1$. (See Slides for Set-up, Base case, Intro)

Slightly different approach from class:

Let $k \geq 0$. We assume for induction that $P(k)$ is true.
This means $2^k + 1 \leq 3^k$
Since $2 < 3$, we can multiply the left side by 2 and the right side by 3:

$$2^{k+1} + 2 \leq 3^{k+1}$$

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Now since $1<2$

$$2^{k+1}+1 < 2^{k+1}+2$$

So

$$2^{k+1}+1 < 3^{k+1}$$

Therefore, $P(k+1)$ is true.