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Goals:

- Describe graphs using adjacency matrices & Lists

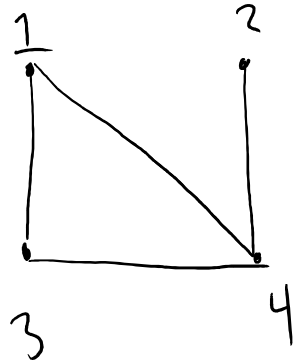
Announcements

- Next week
- PA2 clarification

Ways to Represent Graphs in Computer

Adjacency Matrix

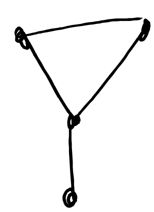
$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$



Store as array A in memory. E.g. $A[3,4]=1$

- Can learn $A[i,j]$ in $O(1)$ time

Which adjacency matrix represents this graph?



$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

A

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

B

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

C

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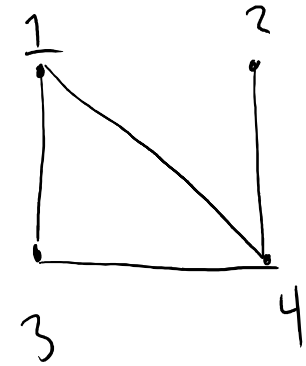
Adjacency List

Vertex	Adjacent Vertices
1	3, 4
2	4
3	1, 4
4	1, 2, 3



Store as an array of lists

Can access A[3], (the list) in $O(1)$ time, but then to go through list takes time $O(L)$ where L is length of list. Can learn A[3].length in $O(1)$ time.

ex: $A[1] = \{3, 4\}$

$A[3, 2] = 4$

$A.\text{length} = 4$

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Edge List

Can represent graph as list of edges, but worstcase time complexity bad for most applications

def: The degree of a vertex is the number of adjacent edges.

def: A vertex v_1 is adjacent to vertex v_2 if connected by an edge

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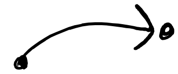
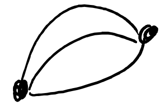
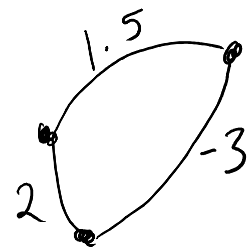
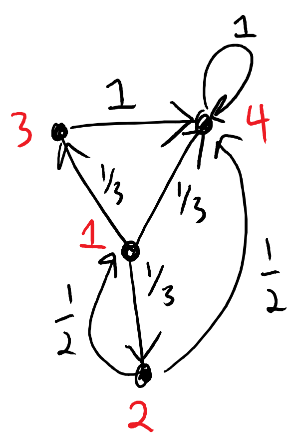
How would you represent a

- directed graph? 
- graph with self-loops? 
- graph with multiedges? 
- graph with weighted edges? 

Using Adjacency Matrix / Adjacency List?

Give representations of this graph using both approaches:



	1	2	3	4
1	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
2	$\frac{1}{2}$	0	0	$\frac{1}{2}$
3	0	0	0	1
4	0	0	0	1

V	List
1	(2, 1/3), (3, 1/3), (4, 1/3)
2	(1, 1/2), (4, 1/2)
3	(4, 1)
4	(4, 1)

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Input: Adj Matrix A for $G=(V,E)$ (undirected, unweighted, no self loops)

Output:

1. $S=0$
2. for $u,v \in V$

   $S \mathrel{+}= A[u,v]$
3. return S

A) $|V|$ B) $|V| \times |V|$ C) $|E|$ D) $\boxed{2|E|}$