## Learning Goals

- Identify statements and predicates
- Translate English predicates to math predicates
- Create proofs using truth tables


## Which are statements?

1. If the input has $x$ bits, this algorithm* uses at most $2 x^{2}$ operations.
2. For all $x>10$, if the input has $x$ bits, this algorithm* uses at most $2 x^{2}$ operations.
3. This sentence (the one you are reading right now) is false.
4. Is QuickSort faster than MergeSort?
*Assume l've given you a description of some particular algorithm.

|  |  | $\begin{array}{\|l\|l\|} \hline \text { If } P \text { then } \\ Q " \end{array}$ | "P if and only $\text { if } Q^{\prime \prime}$ | "P and Q" | "P or Q" | "not $P^{\prime \prime}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $P$ | $Q$ | $\boldsymbol{P} \rightarrow \boldsymbol{Q}$ | $\boldsymbol{P} \leftrightarrow \boldsymbol{Q}$ | $P \wedge Q$ | $\boldsymbol{P} \vee \mathbf{Q}$ | $\neg P$ |
| T | T | T | T | T | T | F |
| T | F | F | F | F | T | F |
| F | T | T | F | F | T | T |
| F | F | T | T | F | F | T |


|  |  | "If $P$ then Q" | " $P$ if and only if $Q^{\prime \prime}$ | "P and Q" | "P or Q" | "not P" |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $P$ | $Q$ | $P \rightarrow Q$ | $\boldsymbol{P} \leftrightarrow \boldsymbol{Q}$ | $P \wedge Q$ | $P \vee Q$ | $\neg P$ |
| T | T | T | T | T | T | F |
| T | F | F | F | F | T | F |
| F | T | T | F | F | T | T |
| F | F | T | T | F | F | T |


|  |  | "If $\boldsymbol{P}$ then <br> $\boldsymbol{Q}$ " | " $\boldsymbol{P}$ if and only <br> if $\boldsymbol{Q}$ " | " $\boldsymbol{P}$ and $\boldsymbol{Q}$ " | " $\boldsymbol{P}$ or $\boldsymbol{Q}$ " | "not $\boldsymbol{P}$ " |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{P}$ | $\boldsymbol{Q}$ | $\boldsymbol{P} \rightarrow \boldsymbol{Q}$ | $\boldsymbol{P} \leftrightarrow \boldsymbol{Q}$ | $\boldsymbol{P} \wedge \boldsymbol{Q}$ | $\boldsymbol{P} \vee \boldsymbol{Q}$ | $\neg \boldsymbol{P}$ |
| T | T | T | T | T | T | F |
| T | F | F | F | F | T | F |
| F | T | T | F | F | T | T |
| F | F | T | T | F | F | T |


|  |  | "If $P$ then Q" | " $P$ if and only if $Q^{\prime \prime}$ | "P and Q" | "P or Q" | "not P" |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $P$ | $Q$ | $P \rightarrow Q$ | $\boldsymbol{P} \leftrightarrow \boldsymbol{Q}$ | $P \wedge Q$ | $P \vee Q$ | $\neg P$ |
| T | T | T | T | T | T | F |
| T | F | F | F | F | T | F |
| F | T | T | F | F | T | T |
| F | F | T | T | F | F | T |


|  |  | "If $\boldsymbol{P}$ then <br> $\boldsymbol{Q} "$ | " $\boldsymbol{P}$ if and only <br> if $\boldsymbol{Q}$ " | " $\boldsymbol{P}$ and $\boldsymbol{Q}$ " | " $\boldsymbol{P}$ or $\boldsymbol{Q}$ " | "not $\boldsymbol{P}$ " |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{P}$ | $\boldsymbol{Q}$ | $\boldsymbol{P} \rightarrow \boldsymbol{Q}$ | $\boldsymbol{P} \leftrightarrow \boldsymbol{Q}$ | $\boldsymbol{P} \wedge \boldsymbol{Q}$ | $\boldsymbol{P} \vee \boldsymbol{Q}$ | $\neg \boldsymbol{P}$ |
| T | T | T | T | T | T | F |
| T | F | F | F | F | T | F |
| F | T | T | F | F | T | T |
| F | F | T | T | F | F | T |


|  |  | $\begin{array}{\|l\|l\|} \hline \text { If } P \text { then } \\ Q " \end{array}$ | "P if and only $\text { if } Q^{\prime \prime}$ | "P and Q" | "P or Q" | "not $P^{\prime \prime}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $P$ | $Q$ | $\boldsymbol{P} \rightarrow \boldsymbol{Q}$ | $\boldsymbol{P} \leftrightarrow \boldsymbol{Q}$ | $P \wedge Q$ | $\boldsymbol{P} \vee \mathbf{Q}$ | $\neg P$ |
| T | T | T | T | T | T | F |
| T | F | F | F | F | T | F |
| F | T | T | F | F | T | T |
| F | F | T | T | F | F | T |


|  |  | $\begin{aligned} & \text { "If } P \text { then } \\ & Q " \end{aligned}$ | $\text { " } P \text { if and only }$ $\text { if } Q^{\prime \prime}$ | "P and Q" | "P or Q" | "not $P^{\prime \prime}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $P$ | $Q$ | $P \rightarrow Q$ | $\boldsymbol{P} \leftrightarrow \boldsymbol{Q}$ | $P \wedge Q$ | $\boldsymbol{P} \vee \mathbf{Q}$ | $\neg P$ |
| T | T | T | T | T | T | F |
| T | F | F | F | F | T | F |
| F | T | T | F | F | T | T |
| F | F | T | T | F | F | T |

Please Memorize (will get practice on Pset 1).
(To speak a new language, you need to memorize some vocab.)

## Truth Table Proof

- Prove $(P \rightarrow Q) \vee(Q \rightarrow R)$ is true using a truth table.
- Explain in words why it is true without a truth table.
-Prove: $(P \rightarrow Q) \leftrightarrow(\neg P \vee Q)$ (can extend original table)


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