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Learning Goals

- Identify statements and predicates
- Translate English predicates to math predicates
- Prove a statement true using truth table proofs

Statements ≈ math sentences

also called proposition

definition: A statement is a declarative sentence that is true or false.

Why Important?

- In C.S. we often want to convince other people something is true or false.

statement

use proof

*assume I've given you a description of the algorithm

Q: Which are statements?

- If the input has x bits, this algorithm* uses at least $2x^2$ operations.
- For all integers $x > 10$, if the input has x bits, this algorithm* uses at least $2x^2$ operations.
- This sentence (the one you are reading right now) is false.
- Is QuickSort faster than MergeSort?

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- If the input has x bits, this algorithm uses at least $2x^2$ operations.

Not a statement. True for $x=0$
False for $x=10$
Depends on x

$\boxed{\text{Predicate}}$

- For all integers $x>10$, if the input has x bits, this algorithm uses at least $2x^2$ operations.

Statement: either true (for all $x>10$)
or false (not all $x>10$ work)

(Quantifying a predicate $\rightarrow$ becomes statement)

- This sentence (the one you are reading right now) is false.

Not a statement: If true, it is false
If false, it is true.
Can't be true or false!

- Is QuickSort faster than MergeSort?

Not a statement: Question is not declarative.

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def: A predicate is a declarative sentence with a variable such that when that variable is assigned a value, the predicate becomes a statement.

(or variables)

ex:

If the input has x bits, this algorithm uses at least $2x^2$ operations.

$\Uparrow$

Call this sentence $P(x)$.

Perhaps: $P(5)$ = false statement

$P(20)$ = true statement

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Proof Goal:

Known true statements $\longrightarrow$ New true statements

One tool: combining statements

I learn. = "P"
I study. = "Q"

} assign letter to basic, atomic statements

Can combine with

$\longrightarrow$

$\longleftrightarrow$

$\wedge$

$\vee$

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Truth Table

P	Q	"If P then Q" $P \to Q$	"P if and only if Q" $P \leftrightarrow Q$	"P and Q" $P \wedge Q$	"P or Q" $P \vee Q$	"not P" $\neg P$
T	T	T	$\boxed{T}$	$\boxed{T}$	T	F
T	F	F	F	F	T	F
F	T	T	F	F	T	T
F	F	T	$\boxed{T}$	F	F	T

Memorize

All possible combinations of T and F

Can think of as Boolean variables

If P is false, $P \to Q$ is true

$\Downarrow$

$\boxed{\text{If my dog has wings, I will be queen of the world.}}$

True statement

True when both are the same

only true if both are true

true if either is true OR BOTH

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What is wrong with the following predicate?

$$x > 5 \land < 10$$

("x is greater than 5 and less than 10")

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What is wrong with the following predicate?

$$x > 5 \wedge < 10$$

("x is greater than 5 and less than 10")

"Grammar" error: On each side of symbols $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$ should be a predicate or statement

$$\boxed{x > 5} \wedge \boxed{< 10}$$

predicate

"Less than 10"
Not a sentence, not a predicate or statement

Should be:

$$(x > 5) \wedge (x < 10)$$

(use parentheses for clarity)

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☆ Be careful when translating English to Math. You can't always be literal.

☆ $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$ are like functions that take 2 statements/predicates as input and give 1 statement/predicate as output

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I will convince you the following is true:

If you eat a raw egg everyday, then you will win the lottery, or if you win the lottery you will lose your job.

P

Q

R

Truth Table Proof

1. Identify and label atomic statements
2. Write statement symbolically

$$(P \to Q) \vee (Q \to R)$$

3. Create truth table. Use it to show statement always true

Q: How many rows will the truth table have?

A) 3 B) 4 C) 6 D) 8

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ATOMIC — Build Up to Complex

8 Rows

P	Q	R	$P \to Q$	$Q \to R$	$(P \to Q) \vee (Q \to R)$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

all "assignments"

Proof: From the truth table, we see

$(P \to Q) \vee (Q \to R)$ is true for any assignment of P, Q, R.

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Truth tables are useful when have few atomic statements (# of rows = $2^{\text{\# of atomic statements}}$)

Instead, can prove by explaining:

Q. Prove $(P \to Q) \vee (Q \to R)$ is true

options

There are 2 cases for Q. Either it is true or false (not both!). If it is true, $P \to Q$ is true. If it is false, $Q \to R$ is true. Either way, at least $P \to Q$ or $Q \to R$ is true, so $(P \to Q) \vee (Q \to R)$ is true.