Name _____

CS200 - Midterm 1 Review Questions

1. Let S be the set of all people who have every lived. Let $G(x, y)$ be the predicate, x is the grandmother of y, for $x, y \in S$. Let $C(x, y)$ be the predicate x and y are cousins for $x, y \in S$. Write the following statements and predicates using math:

   (a) "All people have at least two grandmothers."

   (b) "Every pair of cousins share a grandmother."

   (c) "None of Person x's cousins are grandmothers "

   (d) (Challenge) "All of Person x's children except one are childless."

2. You meet a group of 50 orcs. You know orcs are either honest or corrupt. Suppose you know that at least one of the orcs is honest. You also know that given any two of the orcs, at least one is corrupt. Let G be the set of orcs, and $D(g)$ is the predicate "orc g is corrupt."

   (a) How many of the orcs are corrupt and how many are honest?

   (b) Express "At least one orc is honest" using math.

   (c) Express "Given any two orcs, at least one is corrupt" using math

3. Consider ways of proving: $14|a$ if and only if $2|a$ and $7|a$.

   (a) How would you start a direct proof of the forward direction?

   (b) How would you start a direct proof of the backwards direction?

   (c) How would you start a contrapositive proof of the forward direction?

   (d) How would you start a contrapositive proof of the backwards direction?

   (e) How would you start a proof by contradiction of the forward direction?

   (f) How would you start a proof by contradiction of the backwards direction?

   (g) In this case it turns out that a direct proof is the best approach in both cases. If you'd like practice, complete the proof. Also, explain why the other approaches are less good.

4. **[11 points]** Prove using induction that program `ProductThing`(n) returns a number less than or equal to n^n for all $n \geq 1$.

Algorithm 1: `ProductThing`(m)

Input : $m \in \mathbb{Z}$ such that $m \geq 1$

Output: Something.

```
// Base Case
if m == 1 then
    return 1;
else
    // Recursive step
    return ProductThing(m − 1) × m
end
```

5. **Sorry I introduced a typo. This is not actually true!!** Prove that if $a, b \in \mathbb{Z}$, then $a^2 - 2b - 2 \neq 0$ using a proof by contradiction (BOP 6A6)