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<u>def</u>: Let $f, g: \mathbb{Z} \to \mathbb{R}^{+/0}$ or $f, g: \mathbb{R} \to \mathbb{R}^{+/0}$, $\mathbb{R}^{+/0} = \{x : x \geq 0 \wedge x \in \mathbb{R}\}$

- Then $\underbrace{f(x) \text{ is } O(g(x))}_{}$ if $\exists\, k, C \in \mathbb{R}^{+}: \forall x \geq k,$

$$f(x) \leq C g(x).$$

"f of x is big-oh of g of x"

"big omega"

- $f(x)$ is $\Omega(g(x))$ if $\exists\, k, C \in \mathbb{R}: \forall x \geq k$

$$f(x) \geq C g(x)$$

- $f(x)$ is $\Theta(g(x))$ if $f(x) = O(g(x))$ and $f(x) = \Omega(g(x))$

big theta

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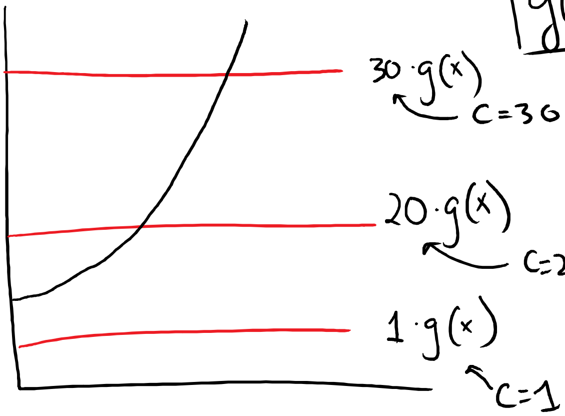
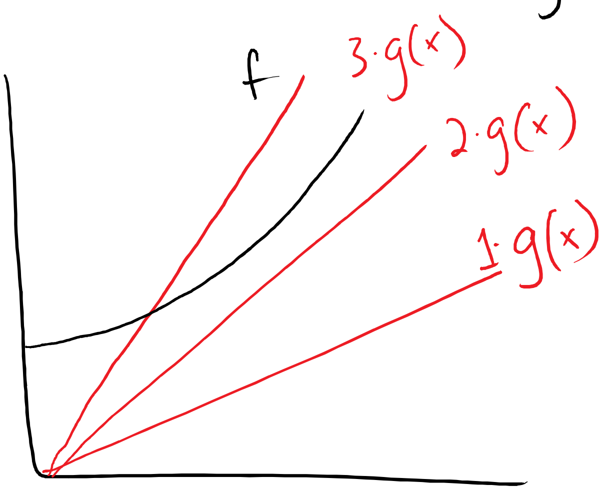
ex: $f(x) = 2x^2 + 10$

$\boxed{g(x) = 1}$



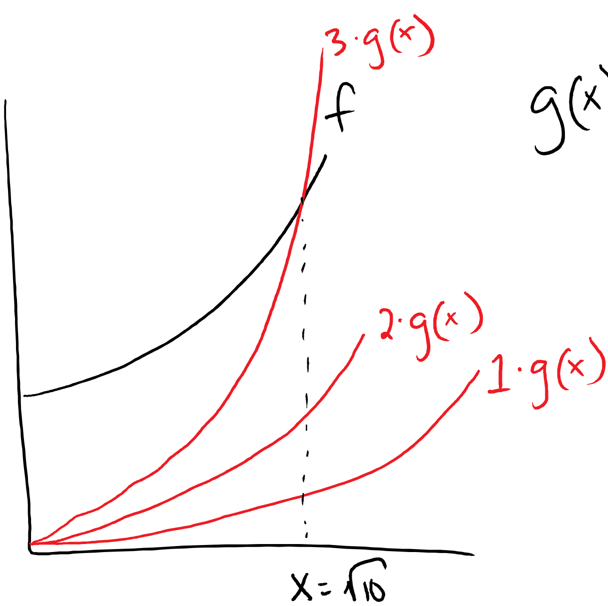
$f(x) \neq O(1)$ because at large x, f always goes above g no matter how big we make C

$g(x) = x$



$f(x) \neq O(x)$ " "

$g(x) = x^2$



$f(x) = O(x^2)$ b/c with $f(x) \leq 3x^2$ for all $x > \sqrt{10}$.

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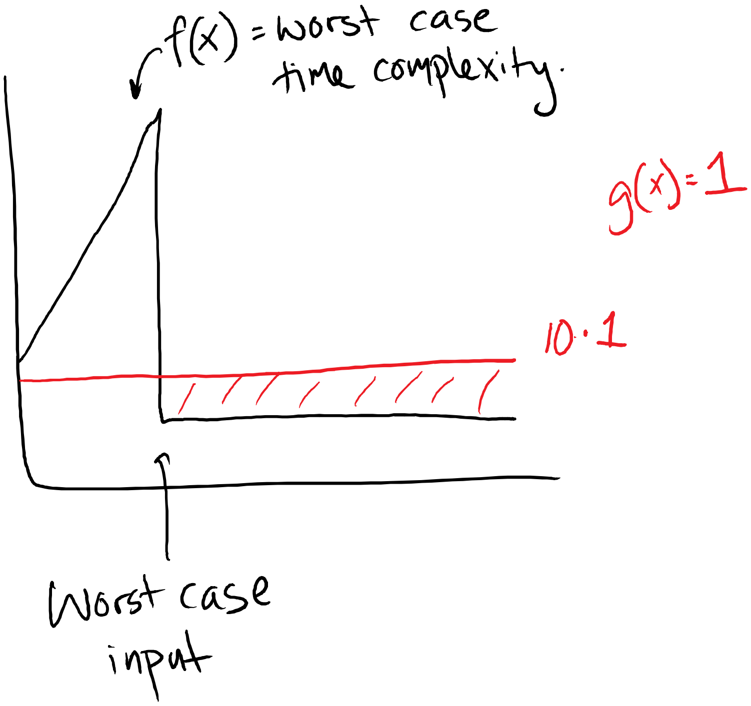

$f(x) = O(1)$

Big-O bound tells you about scaling, not the actual worst case.

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Prove: $2x^2+10 \neq O(x)$

Assume for contradiction that

$2x^2+10 = O(x)$

This means $\exists\, k, C \in \mathbb{R}^+$ s.t. $\forall\, x \geq k$, $2x^2+10 \leq Cx$

Now let $y > k$ and $y > C$ then

$$2y + \frac{10}{y} < C \quad b/c \;\; y > 0,\; y > k.$$

But because $y > C$, we have

$$2y + \frac{10}{y} > 2C, \quad \text{a contradiction.}$$