

Goals

- Translate predicates
- Apply de Morgan's rule

De Morgans Rules (Rules to distribute \neg)

$$\neg(P \wedge Q) \neq \neg P \wedge \neg Q$$

CAUTION!
DON'T DISTRIBUTE!

Instead

$$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$$

$$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$$

- Distribute
 - flip $\wedge \leftrightarrow \vee$

Also

$$\neg \forall x \in A, P(x) \equiv \exists x \in A : \neg P(x)$$

$$\neg \exists x \in A : P(x) \equiv \forall x \in A, \neg P(x)$$

- Distribute
 - Switch
 - $\exists \leftrightarrow \forall$

For example:

double negative
↓
doesn't change

de Morgan
↓

$$\forall x \in S, P(x) \equiv \neg \neg \forall x \in S, P(x) \equiv \neg \exists x \in S : \neg P(x)$$

Q: Which is logically equivalent to:

$$\neg \exists x \in \mathbb{Z} : x^2 = x \wedge x > 1$$

A: $\forall x \in \mathbb{Z}, x^2 \neq x \vee x \leq 1$

B: $\forall x \in \mathbb{Z}, x^2 \neq x \wedge x \leq 1$

C: $\exists x \in \mathbb{Z} : x^2 \neq x \vee x \leq 1$

D: $\exists x \in \mathbb{Z} : x^2 \neq x \vee x \leq 1$

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De Morgan's Law:

$$\Rightarrow \forall x \in \mathbb{Z}, \neg (x^2 = x \wedge x > 1)$$

De Morgan again

$$\Rightarrow \forall x \in \mathbb{Z}, (\neg x^2 = x \vee \neg x > 1)$$

Rewrite
 $\Rightarrow \forall x \in \mathbb{Z}, (x^2 \neq x \vee x \leq 1)$