

Goals

- Describe sets using set-builder notation
- Translate quantified predicates

Set Builder Notation

$B = \{ f(x) : P(x) \}$ = the set of x where $P(x)$ is true, with $f(x)$ applied to each element

↑
 function of x

↑
 predicate of x

ex: $A = \{ x^2 : x \text{ is even} \} = \{ 0^2, 2^2, 4^2, 6^2, \dots, (-2)^2, (-4)^2, \dots \}$

Repeats not included
 ✓ ↓

$= \{ 0, 4, 16, 36, \dots \}$

Other ways to represent same set:

$$A = \left\{ x : \frac{\sqrt{x}}{2} \in \mathbb{Z} \right\}$$

Q: Others?

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$$A = \left\{x : \frac{\sqrt{x}}{2} \in \mathbb{Z}\right\}$$

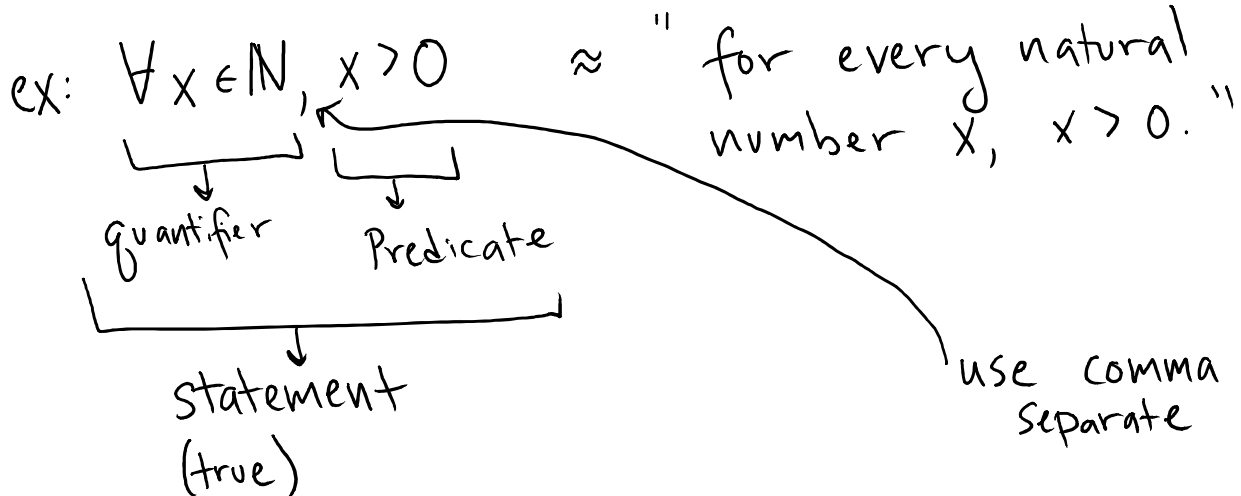
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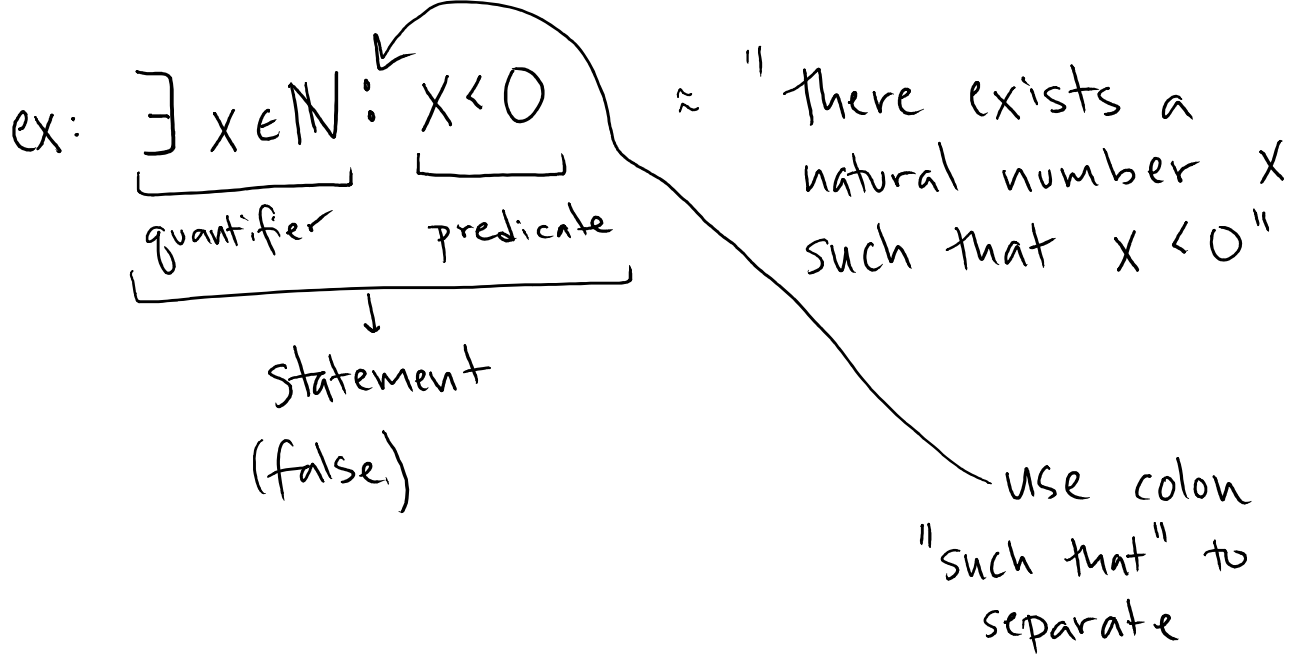
$$A = \{(2x)^2 : x \in \mathbb{Z}\}$$

Quantifiers : turn predicates into statements

Universal Quantifier : \forall means "for all", "every"



Existential Quantifier : \exists means "there exists" "there is"



Q: Let $M(x, y)$ be the predicate "x is y's parent." Let S be the set of all people who have ever lived.

True or false:

$$\forall x \in S, \exists y \in S: M(x, y)$$

True or false?

- $\forall x \in S, \exists y \in S: M(y, x)$
- $\exists x \in S, \forall y \in S: M(x, y)$
- $\exists x \in S, \forall y \in S: M(y, x)$

Q: Let $M(x,y)$ be the predicate that x is y 's parent. Let S be the set of all people.

True or false:

$$\forall x \in S, \exists y \in S: M(x,y)$$

False:

"Every person has a child"

To determine:

- Imagine you are playing a game against an opponent.
- Opponent: gets to choose \forall , wins if makes false
- You: get to choose \exists , win if make true
- Both: can choose based on previous person's choice
- Who wins? \rightarrow True (You)
 \rightarrow False (Opponent)

ex: $\forall x \in S, \exists y \in S: M(x,y)$

↑↑
opponent
can choose any
person. Best strategy:
choose x to be
someone without a kid

↙↘ If opponent chooses someone with
no kid, we can't find a y to
be x 's kid. We lose! Statement
is false.

Q: Let $M(x,y)$ be the predicate that x is y 's parent. Let S be the set of all people.

True or false:

$$\forall x \in S, \exists y \in S: M(x,y)$$

False:

"Every person has a child"

Other orders

- $\forall x \in S, \exists y \in S: M(y,x) \equiv$ "Every person has a parent." True
- $\exists x \in S, \forall y \in S: M(x,y) \equiv$ "One person is the parent of all people" False
- $\exists x \in S, \forall y \in S: M(y,x) \equiv$ "One person is the child of all people" False

* Quantifier must have domain

~~$$\forall x, x > 0$$~~

$$\forall x \in \mathbb{Z}, x > 0$$

\downarrow
 domain = a set

Tips for successful translation

- "All natural numbers less than 5 divide 12"
subset of \mathbb{N}

$$\forall x \in \mathbb{N}, \underbrace{x < 5} \rightarrow \underbrace{x | 12}$$

condition on subset "x divides 12"

Common: $\forall x \in _, _ \rightarrow _$

- Can combine 2 of same type: $\forall x, y \in \mathbb{N}, \dots$

- $\exists y \in S : M(x, y) \equiv R(x)$
 - ← this is a predicate with variable x
 - ↑ "y is quantified"
 - ↑ "x is not quantified"

$$R(\text{Prof K}) = \text{True} \quad (\text{I have kid.})$$

$$R(\text{Kora}) = \text{False} \quad (\text{Kora doesn't have a kid.})$$

- Any non-quantified variables become inputs to predicate.
- Statements: all variables must be quantified.