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Q: Can we use induction to prove this algorithm is correct (see slides)?

A: Yes

B: No

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Recursive algorithm that sums list

SUM2(| 1 | 8 | 3 | 2 | 9 |) ← 5 elements in list

recursive call ↙ ↘ recursive call

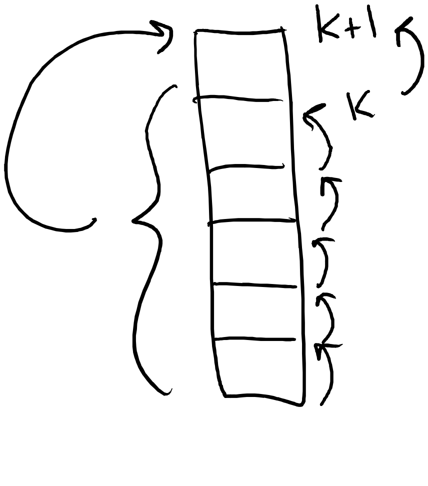
SUM2(| 1 | 8 |) + SUM2(| 3 | 2 | 9 |)

↑ size 2 ↑ size 3

Standard Induction

We want

Idea with Strong Induction



When you are trying to prove $k+1^{th}$ level, why limit youself to only using k^{th} level. Can assume all Predicates from base to k are true ex:

Base case $P(1)$ ✓

$P(1) \to P(2)$ $P(2)$ ✓

To prove $P(3)$, both $P(1)$ and $P(2)$ are true

Can do $(P(1) \wedge P(2)) \longrightarrow P(3)$ $P(3)$

No we know $P(1), P(2), P(3)$ are true. We can use all to prove $P(4)$.

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Strong Induction Proof Structure

Set-up

Let P(n) be the predicate ______________ We will prove P(n) is true for all $n \geq$ b.c.

base case

Base-case

Base-case: We prove P(b.c.) is true. . . . (Sometimes need multiple base cases) P(0), P(1))

Inductive case

Inductive case: Let $k \geq$ b.c.. Assume for strong induction that [P(j) is true for all j such that b.c. $\leq j \leq k$.]

$\boxed{\forall j \in \mathbb{Z}, \text{b.c.} \leq j \leq k \rightarrow P(j)}$

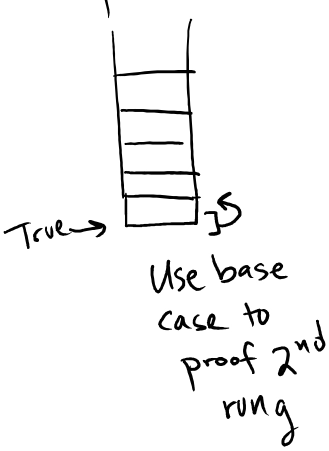
Explain

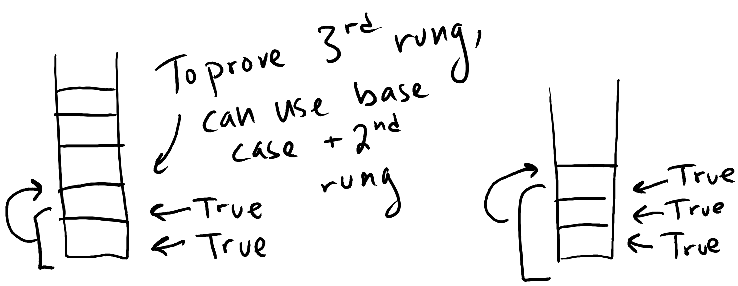
Therefore P(k+1) is true

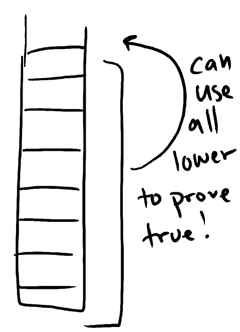
Conclusion

By strong induction, we conclude P(n) is true for all $n \geq$ b.c.

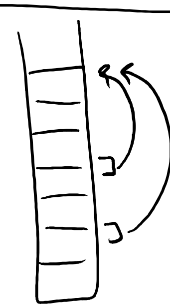
Metaphor:





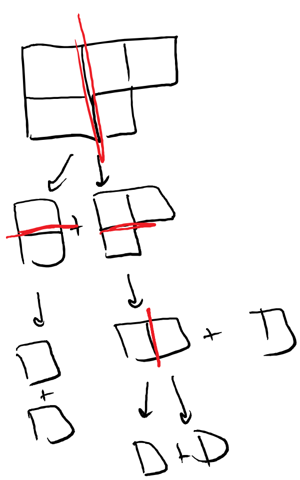


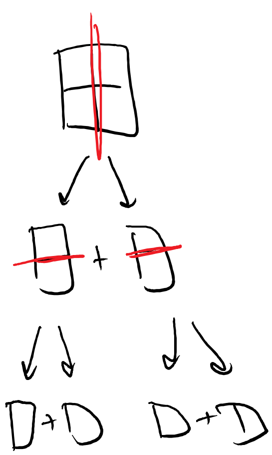
Usually: just need a couple of lower rungs, but we assume all lower rungs are true to make things simpler



Strong Induction

Q: Suppose you have a bar of chocolate containing n small joined squares. How many times do you have to break the chocolate along a row or column before you have n separate squares?





A) $n-1$

B) n

C) Depends on original shape

D) I want chocolate

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Q: Prove it takes $n-1$ breaks to reduce an n-square chocolate bar to n individual squares.

↓

A: Let $P(n)$ be the predicate " ." We will prove via strong induction that $P(n)$ is true for $n \in \mathbb{N}$, $n \geq 1$.

Base case: When you have a 1-square chocolate bar, it requires 0 breaks to create 1 individual squares, so $P(1)$ is true.

Inductive Case:
Let $k \geq 1$. We assume for strong induction that $P(j)$ is true for $1 \leq j \leq k$. We will prove $P(k+1)$ is true. Since $k+1 > 1$, we can break the chocolate into two pieces, one with a squares, and one with b squares, where $a+b = k+1$, and $1 \leq a \leq k$, and $1 \leq b \leq k$. Using our inductive assumption, it requires $(a-1)$ breaks to separate the first piece and $(b-1)$ breaks to separate the second. Adding up all the breaks, we have

$$(a-1) + (b-1) + 1 = a+b-1 = k$$

total breaks. Thus $P(k+1)$ is true.

Therefore, by strong induction, $P(n)$ is true.