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$K(n)$ = # of n-digit #'s with an even # of 0s.
Create a recurrence relation for $K(n)$

Strategy

1. Recurrence ← Use counting rules
2. Base case

* Think about options for final digit

$$K(n) = \begin{bmatrix}\text{even \# of 0's} \\ \text{end with 0}\end{bmatrix} \overset{\text{or}}{+} \begin{bmatrix}\text{even \# of 0's} \\ \text{end with not 0}\end{bmatrix}$$

$\underbrace{\_\ \_\ \_\ \_\ \_\ \_}_{n-1}\ 0$ $\qquad$ $\underbrace{\_\ \_\ \_\ \_\ \_\ \_}_{n-1}\ \frac{1,2,3,4,\ldots}{9}$ ← 9 options

Need this part to have odd # of 0's.

Need this part to have even # of 0's & n-1 digits $\Rightarrow K(n-1) \rightarrow \boxed{9K(n-1)}$

$$\begin{bmatrix}\text{\# n-1 digits with} \\ \text{odd \# of 0's}\end{bmatrix} = \boxed{\begin{matrix}\text{\# n-1 digit} \\ \text{\#'s}\end{matrix}} - \boxed{\begin{matrix}\text{\# n-1 digit \#'s} \\ \text{with even \# of 0's}\end{matrix}}$$

$$\boxed{10^{n-1} - K(n-1)}$$

$$K(n-1) = 10^{n-1} - K(n-1) + 9K(n-1) = 10^{n-1} + 8K(n-1)$$

Base case: $K(1) = 9$ (0 is an even #)