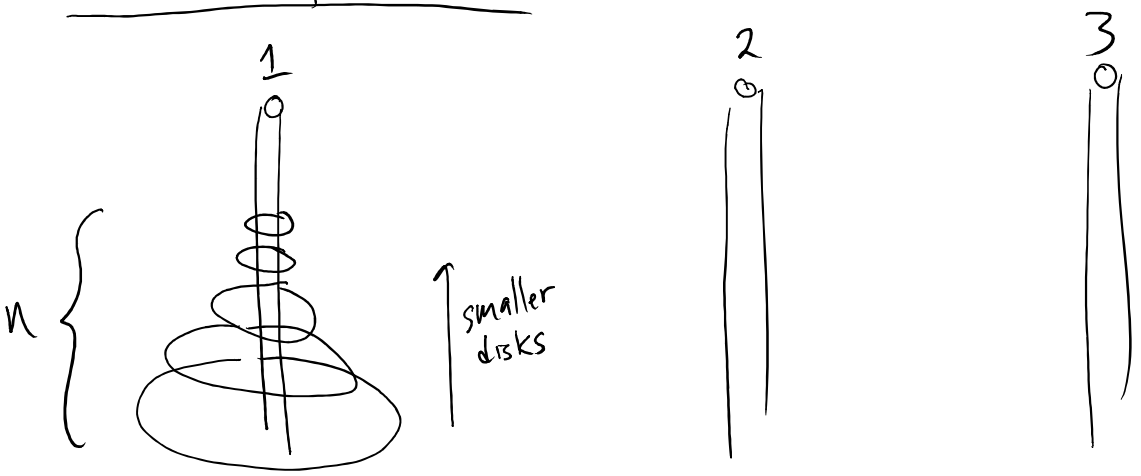
S. KIMMEL

Another example...

Towers of Hanoi



Task: Move Tower from 1 to 3 without putting larger disk on top of smaller.

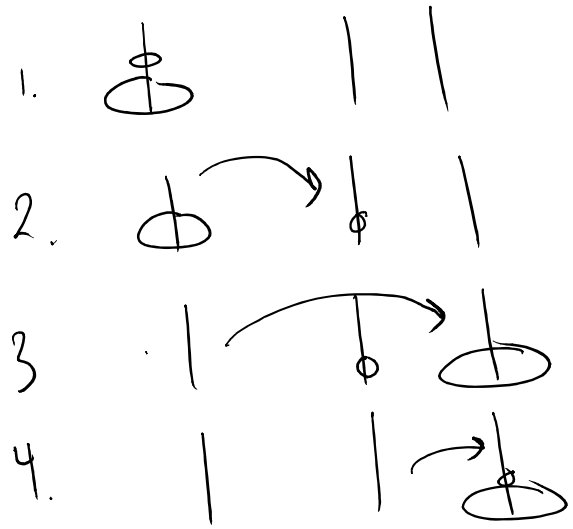
Q: How many moves do you need?

ex 1.

2.

3.

4.



n=2 → 3 moves

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Q: Which of the following recurrence relations is correct if $T(n)$ = # of moves for n disks?

A) $T(1) = 1$; $T(n) = 2 + T(n-1)$

B) $T(2) = 1$; $T(n) = 2T(n-1)$

C) $T(1) = 1$; $T(n) = 2T(n-1)$

D) $T(1) = 1$; $T(n) = 2T(n-1) + 1$

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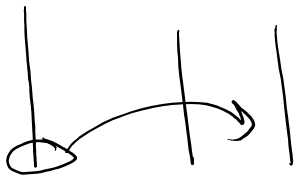
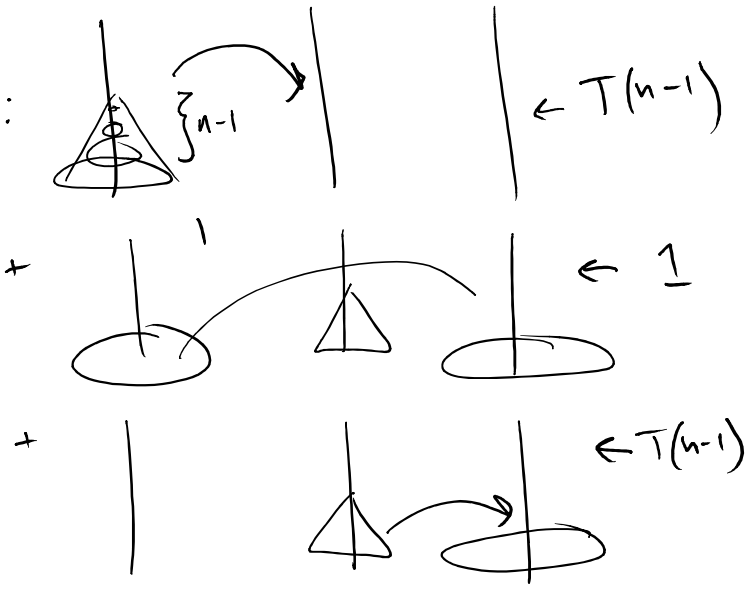
Answer D:

$T(1) = 1$



$T(n)$:



Think about how you could solve using the fact that to move n-1 disks takes time $T(n-1)$

Solving a Recurrence Relation (iterative method)

1. Plug in several times on Right hand side using recurrence.

(Towers of Hanoi)

Recurrence level (k)

1 $\Rightarrow$ $T(n) = 2T(n-1) + 1$

2 $\Rightarrow$ $T(n) = 2[2T(n-2)+1]+1 = 4T(n-2)+2+1$

3 $\Rightarrow$ $T(n) = 4[2T(n-3)+1]+2+1 = 8T(n-3)+4+2+1$

2. Guess the pattern as a function of k

ex: Towers of Hanoi

$$T(n) = 2^k T(n-k) + \sum_{i=0}^{k-1} 2^i$$

3. Find value of k such that $T(\cdot)$ on right hand side is base case, plug in to solve

ex: Base case is $T(1)$. Want $n-k=1$

$\Downarrow$ solve for k

$$k = n-1$$

Plug $k=n-1$ into expression from part 2

$$T(n) = 2^{n-1} T(n-(n-1)) + \sum_{i=0}^{n-2} 2^i$$

$$= 2^{n-1} T(1) + \sum_{i=0}^{n-2} 2^i$$

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$$T(n) = 2^{n-1}\underbrace{T(1)}_{=1} + \underbrace{\sum_{i=0}^{n-2} 2^i}$$

$T(1) = 1$ because base case

↓ geometric series

$$\sum_{i=0}^{g} r^i = \begin{cases} \frac{1-r^{g+1}}{1-r} & \text{if } r \neq 1 \\ g+1 & \text{if } r = 1 \end{cases}$$

$$\frac{1-2^{n-1}}{1-2} = 2^{n-1} - 1$$

$$T(n) = 2^n - 1$$

Note: Iterative method is not a proof... need to turn into inductive proof to fully prove. I will not ask you to do this.

Tip: # of strings with some property? (addition)
Think about options for final digit of string, use subtraction rule

ends in 0 or ends in 1