Goals

- Recognize + interpret proof language
- Write a direct proof

<u>Proof</u>: Argument showing statement is true

Proof Language

- Introduce variables
  - "Let a be an integer" $\equiv \forall a \in \mathbb{Z}$
  - "Some integer K" $\equiv \exists K \in \mathbb{Z}$
- Translate English/Math
  - "Namely," "That is"
- Combine statements
  - "We get," "We have," "Then
- Deduction
  - "Therefore" "Thus" "This means"

- Starting truths / assumption "hypothesis"
  - "Suppose" "assume"

<u>Other</u>:

- Full sentences
- Complex math seperated
- Chain of equalities $A = B = C = D$ is represented by

$$\begin{aligned} A &= B \\ &= C \\ &= D \end{aligned}$$

All prove: $\forall a, b \in \mathbb{Z},\ 2 \mid ab \rightarrow (2 \mid a \vee 2 \mid b)$

1. Contrapositive
2. Contradiction
3. Proof by cases with Direct proof

Direct Proof

Use: Prove $P \to Q$

Structure:

Assume P. Explain, explain, explain. Therefore Q.

Use: Prove $\forall x \in S,\ P(x) \to Q(x)$

Structure

Let $x \in S$, and assume $P(x)$.
Explain, explain, explain.
Therefore $Q(x)$

Examples of explain:
- convert English to math
- combine equations
- manipulate equations
- deduce

Sometimes $\forall x \in S$ is implied, obvious.
In that case, can leave off "Let $x \in S$"

ex: "Let a,b be integers, and suppose a,b are odd"

$\Downarrow$ Also OK

"Suppose a,b are odd" (the fact that they are integers is implied)

Q: Prove: If $a|b$ and $b|c$ then $a|c$.

$(a|b \equiv \exists e \in \mathbb{Z} : ae = b)$

Hint: If don't know where to start, use math.

Q: Prove: If $a|b$ and $b|c$ then $a|c$.
$(a|b \equiv \exists e \in \mathbb{Z} : ae = b)$

Let $a, b, c \in \mathbb{Z}$. Assume $a|b$ and $b|c$. This means $\exists$ $e, f \in \mathbb{Z}$ such that $b = ae$ and $c = bf$. Then

$$c = bf = (ae)f = a(ef).$$

That means $a|c$, since $c = ak$, for $k = ef$, an integer.

☆ Can leave off "Let $a, b, c \in \mathbb{Z}$". Implied by "Assume $a|b$ and $b|c$"