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Calculating Probability

What is the probability of (2 dice sum to 9)?

1. Define the sample space

think big!

set of all possible outcomes

$S = \{(1,1), (1,2), (1,3) \ldots (6,6)\}$

$= \{1,2,3,4,5,6\}^2$

2. Define the event

subset of sample space you care about

$E = \{(6,3), (3,6), (5,4), (4,5)\}$

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3. Ask: are all elements of sample space equally likely?

Yes → $Pr(E) = \frac{|E|}{|S|}$

No → $Pr(E) = \sum_{i \in E} Pr(i)$

Probability of outcome i

Sum over all elements in E

Note $\sum_{i \in S} Pr(i) = 1$

every outcome is equally likely

$|E| = 4$

$|S| = 6 \times 6$ (product rule)

$Pr(\text{2 dice add to 9}) = \frac{4}{36} = \frac{1}{9}$

Useful fact: $Pr(S - E) = 1 - Pr(E)$

$S - E$: not E

(sometimes easier to calculate not E)

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Q: Lottery where a 4-digit number is chosen randomly. What is the size of the sample space?

A) 40 B) 10,000 C) $\binom{10}{4}$

Use product rule!

$10 \cdot 10 \cdot 10 \cdot 10$

Suppose you win some money if you get 3 of 4 numbers matching.

What is the probability you don't win if you buy 1 ticket with # 1313.

1. Sample Space: $\{1,2,3,\ldots,10\}^4$
2. Event (I'll calculate probability of winning and then do $1 - Pr(E)$.)

   $E = \{X : X \in \{1,2,\ldots,10\}^4 \wedge X \text{ agrees with 1313 in at least 3 positions}\}$
3. Use formula $|E|/|S|$ b/c all outcomes are equally likely

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$|E| =>$ agree in all 4 positions or agree in exactly 3 position

$\underline{1}$

one position must differ. Choose which position and Choose how differs

4 options * 9 options

$\underline{1} + 4 \cdot 9 = 37$

$$Pr(\bar{E}) = 1 - \frac{37}{1000}$$

$$= .9963$$

for example

1313
2313
3313
4313
5313
6313
7313
8313
9313

9 options

Differ from 0313 in first position