S. KIMMEL

Permutations + Combinations

Permutation Warm-Up

Q: There are 10 singles left in Coffrin and you and 2 friends want to pick 3 of them. How many ways could you choose rooms.

A) 30 B) 300 C) 720 D) 1000

Combination Warm-Up

Q: There are 10 singles left in Coffrin and you and 2 friends want to pick 3 of them. Suppose you just want to pick 3 rooms now, and you'll figure out who will stay where later. How many ways could you pick 3 rooms?

A) $\frac{720}{6}$ B) $\frac{720}{3}$ C) $\frac{720}{2}$ D) 720

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Permutations + Combinations

Permutation Warm-Up

Q: There are 10 singles left in Coffin and you and 2 friends want to pick 3 of them. How many ways could you choose rooms.

A) 30 B) 300 $\boxed{\text{C) } 720}$ D) 1000

Answer: using product rule $\underbrace{10}_{\text{options for me}} \cdot \underbrace{9}_{\text{options for friend 1}} \cdot \underbrace{8}_{\text{options for friend 2}} = 720$

Combination Warm-Up

Q: There are 10 singles left in Coffin and you and 2 friends want to pick 3 of them. Suppose you just want to pick 3 rooms now, and you'll figure out who will stay where later. How many ways could you pick 3 rooms?

$\boxed{\text{A) } \frac{720}{6}}$ B) $\frac{720}{3}$ C) $\frac{720}{2}$ D) 720

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We know 720 ways if care about order.

If care about order, thes are all different

$$\left\{ \begin{matrix} (2,3,5), (2,5,3), (3,2,5), (3,5,2) \\ (\underset{\substack{\uparrow \\ \text{My pick}}}{5}, \underset{\substack{\uparrow \\ \text{Friend 1 pick}}}{2}, \underset{\substack{\uparrow \\ \text{Friend 2 pick}}}{3}), (5,3,2) \end{matrix} \right.$$

But if don't care about order, these are all the same. $\{2,3,5\}$

$\Rightarrow$ Over counting by a factor of 6 for each set!

$720/6 = 120$

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We will learn rules (like product + subtraction rules) to handle these types of situations

$P: \mathbb{N} \times \mathbb{N} \to \mathbb{Z}$

$C: \mathbb{N} \times \mathbb{N} \to \mathbb{Z}$

<u>Cartesian product</u>:

Sets $A, B, C, \ldots K$

Then $A \times B \times C \times \ldots K = \{ \underbrace{(a, b, c \ldots k)}_{}^{\text{k-tuple}} : a \in A \wedge b \in B \wedge c \in C \wedge \ldots \wedge k \in K \}$

e.g. $S = \{a, b, c, d\}$ then $(a, c) \in S \times S$

Also $\{a, b, c, d\}^3 = S \times S \times S$ e.g. $(c, c, a) \in S \times S \times S$

Q: Which of the following is in $\mathbb{N} \times \mathbb{N}$

A) $\{1, 2\}$ B) $\{1, 1\}$ C) $(0, 2)$, D) $(1, 1)$

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A) not ordered

B) Not a valid expression

C) $0 \notin \mathbb{N}$

D) ✓

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$P: \mathbb{N} \times \mathbb{N} \to \mathbb{Z}$

$C: \mathbb{N} \times \mathbb{N} \to \mathbb{Z}$

no repeats

$P(n,k)$ = # of ways to pick k distinct elements from a set of n elements, if the order matters.

(k-permutation)

"n pick k"

"$_nP_k$"

no repeats

$C(n,k)$ = # of ways to pick k distinct elements from a set of n elements, if the order doesn't matter

(k-combination)

"n choose k"

"$_nC_k$"

"$\binom{n}{k}$"

ex: $P(10,3) = 720$ (roommate problem Choose rooms now)

$C(10,3) = 720/6$ (roommate problem choose rooms later)

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Q: What is the mathematical formula for $P(n,k)$?
For $C(n,k)$?

(Randomly choose person to explain)

$m! = m \cdot (m-1) \cdot (m-2) \cdot (m-3) \cdots 3 \cdot 2 \cdot \underline{1}$

$0! = \underline{1}$

Use product rule!

A. $P(n,k) = \underbrace{n}_{\substack{\text{choices}\\ \text{for 1st}\\ \text{element}}} \cdot \underbrace{(n-1)}_{\substack{\text{choices}\\ \text{for 2nd}\\ \text{element}}} \cdot \underbrace{(n-2)}_{\substack{\text{choices}\\ \text{for 3rd}\\ \text{element}}} \cdots \underbrace{(n-k+1)}_{\substack{\text{choices}\\ \text{for kth}\\ \text{element}}} = \dfrac{n!}{(n-k)!}$

$C(n,k) = \dfrac{P(n,k)}{\#\text{ of orderings of } k \text{ elements}}$ ← "permutations" $= \dfrac{P(n,k)}{P(k,k)} = \dfrac{n! \; 0!}{(n-k)! \; k!} = \dfrac{n!}{(n-k)! \cdot k!}$