

CS200 - Problem Set 1

Please read the syllabus sections on problem sets and honor code before starting this homework. In particular, you should write the names of anyone you worked with on your submission.

1. Please read (at least) Sections 1, 5.5, and 8 of “[I Don’t Code All Day](#)” (click on link on the electronic pdf). Then answer at least one of the following questions. In your response, please be sure to write in the first person about your own experiences, feelings, and/or biases. We will discuss this article and your responses in class on September 18.
 - (a) Did the words of these students resonate with you? Why or why not?
 - (b) Inside or outside of Middlebury, in what ways are these stereotypes reinforced?
 - (c) What types of situations or interactions have helped you to *dispel* these stereotypes?
2. For each of the following sentences, decide whether it is a statement, predicate, or neither, and explain your answer.
 - (a) Call me Ishmael.
 - (b) The universe is supported on the back of a giant tortoise.
 - (c) x is a multiple of 7.
 - (d) The next sentence is true.
 - (e) The preceding sentence is false.
3. Simplify each of the following expressions, where T and F are the Boolean constants *true* and *false*, and p is a variable that represents either *true* or *false* (but not both). For example, consider: $T \wedge p$. If $p \equiv T$, then $T \wedge p \equiv T$, but if $p \equiv F$, then $T \wedge p \equiv F$. Thus $T \wedge p$ takes the same value as p , so we say $T \wedge p \equiv p$. Hint: each answer is one of p , $\neg p$, T , or F .
 - (a) $F \wedge p$
 - (b) $T \vee p$
 - (c) $F \vee p$
 - (d) $T \rightarrow p$

(e) $p \rightarrow p$

(f) $p \vee \neg p$

(g) $p \rightarrow F$

4. Translate the following English sentences into math symbols. (You answers should involve a \rightarrow symbol. Think carefully about what expression goes on either side of the arrow. If you are confused about this, see Section 2.3 of Book of Proof.)

(a) x is equal to 4 or 6 whenever y is greater than 2.

(b) z is at most 5 only if k is at least 10.

5. What can you deduce from the following true statements? Please write the simplest new true statement possible.

$$P \rightarrow R$$

$$Q \rightarrow R$$

$$P \vee Q$$

∴

(1)

6. For the following questions, I expect you to figure out some set notation on your own. I've given you a table (below) of all the relevant "vocab words" with math definitions. (See also, e.g. [DMOI Set Introduction](#).) Pay close attention to inputs and outputs. You will need to memorize these symbols.

symbol	“inputs”	“output”	meaning
$\{a, b, c, \dots\}$	elements a, b, c, \dots	set	“set containing the elements a, b, c, \dots (roster notation)
$\{a : P(a)\}$	elements a , predicate P	set	“the set of elements a such that $P(a)$ is true” (set-builder notation)
$a \in A$	element a , set A	predicate / statement	“ a is an element of the set A ”
$a \notin A$	element a , set A	predicate / statement	“ a is not an element of the set A ”
$A \subseteq B$	sets A, B	predicate / statement	“ A is a subset of B ” $a \in A \rightarrow a \in B$
$A \subset B$	sets A, B	predicate / statement	“ A is a proper subset of B ” $(a \in A \rightarrow a \in B) \wedge A \neq B$
$A \cap B$	sets A, B	set	“the intersection of A and B ” $A \cap B = \{a : a \in A \wedge a \in B\}$
$A \cup B$	sets A, B	set	“the union of A and B ” $A \cup B = \{a : a \in A \vee a \in B\}$
$A - B$ (or $A \setminus B$)	sets A, B	set	“ A minus B ” $A - B = \{a : a \in A \wedge a \notin B\}$
$A \times B$	sets A, B	set	“the Cartesian product of A and B ” $A \times B = \{(a, b) : a \in A \wedge b \in B\}$
\bar{A}	set A	set	“the complement of A ” $\bar{A} = \{a : a \notin A\}$ (there is an assumed “universe” set U , so really we mean $\bar{A} = \{a : a \notin A \wedge a \in U\}$)
$ A $	set A	integer	“the size (cardinality) of A ” number of elements in A

(a) **This problem is moved to PSet 2**

Describe the following sets in roster notation (list the first few elements). If the set is also “famous” give its symbol.

- i. $A = \{2^x : x \in \mathbb{N}\}$
- ii. $B = \{x : x \text{ is even and } x \in \{1, 3, 5\}\}$
- iii. $C = \{x : x \text{ is greater than zero and odd}\}$

(b) **This problem is moved to PSet 2**

Write the following in set-builder notation using as concise and as mathematical notation as possible

- i. $\{2, 4, 6, 8, 10, 12\}$
- ii. $\{1, 3, 5, 7, 9, 11, \dots\}$
- iii. $\{1, 4, 9, 16, 25, 36, \dots\} \cap \{2, 4, 6, 8, 10, \dots\}$

- iv. $\{1, 4, 9, 16, 25, 36, \dots\} \cup \{2, 4, 6, 8, 10, \dots\}$
- v. **Challenge** $\overline{\{1, 4, 9, 16, 25, 36, \dots\} \cup \{2, 4, 6, 8, 10, \dots\}}$ where the universe is \mathbb{N} ,

(c) **This problem is moved to PSet 2**

Let $A = \{1, 2\}$ and $B = \{1, 2, \{3, 4\}\}$

- i. What is $A \times B$? (Use roster notation.)
- ii. What is $|A \times B|$?
- iii. Is $A \subset B$?
- iv. Is $A \subseteq B$?
- v. Is $A \subset A$?
- vi. What is $A - B$?
- vii. What is $A \cup B$?
- viii. What is $A \cap B$?
- ix. If B is the universe, what is \bar{A} ?

(d) **This problem is moved to PSet 2**

Which of the following are the empty set:

- i. $\{x : x \text{ is odd and } 7 < x < 9\}$
- ii. $\{0\}$
- iii. $\{\emptyset\}$
- iv. $\mathbb{Z} \cap \mathbb{Q}$

(e) **This problem is moved to PSet 2**

Let A and B be sets with $|A| = |B|$ such that $|A \cup B| = 7$ and $|A \cap B| = 3$. What is $|A|$? Explain.

(f) Find sets A and B such that $A \subset B$ and $A \in B$.

(g) Does the empty set contain itself?

7. **Challenge** (Taken from Joe Lau and Jonathan Chan's [Knights and Knaves](#) with some name changes under a [CC BY-NC-SA 4.0 licence](#). There are plenty more problems on their website if you enjoy this!)

You are on an island inhabited by knights and knaves. Knights always tell the truth and knaves always lie. You meet three people on the island: Layla, Juan, and Edgar. Layla says that Juan would tell you that Edgar is a knight. Juan says that Edgar is a knight. Edgar says, "Layla is not a knave." Can you tell for sure who is a knight and who is a knave?

8. How long did you spend on this homework?