Goals

- Recognize common translation mistakes
- Write inductive proofs

Announcements
- Exam info
  - Everything up to induction
  - Not strong induction

Translation

S = all living people
$h(s)$ = height of person s
$T(k)$ = Person k is taller than every other person

What is wrong with the following translation:

$$\forall s \in S, \exists k \in S: s \neq k \wedge h(k) > h(s)$$

Style Disclaimer

My proof “recipes” are a guide, they are not law.

As you read more proofs in textbooks, you will see that there is some flexibility in the style within each type of proof. If you feel comfortable, you can vary your proof style.

Inductive Proof Recipe:

- Set-up (need a predicate $P(n)$)
- Base Case
- Inductive Case (assume $P(k)$)
- Conclusion

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Prove: $2^n - 1 \leq 3^n$ for all $n \geq 0$.

Set-up and Base Case

- Let $P(n)$ be the predicate that $2^n - 1 \leq 3^n$. We will prove via induction that $P(n)$ is true for all $n \geq 0$.
- Base case: $P(0)$ is true because $2^0 - 1 = 0$ while $3^0 = 1$, so $2^0 - 1 \leq 3^0$.

Inductive Step

Inductive step: Let $k \geq 0$. We assume for induction that $P(k)$ is true. This means we assume that

$$2^k - 1 \leq 3^k.$$

Multiplying both sides by 2 and then adding 1, we get

$$2^{k+1} - 1 \leq 2 \times 3^k + 1.$$

Now since $k \geq 0$, then $1 \leq 3^k$, so

$$2^{k+1} - 1 \leq 2 \times 3^k + 3^k = 3^k(2+1) = 3^{k+1}.$$

Therefore $P(k+1)$ is true.

Conclusion

- Therefore, by induction on n, $P(n)$ is true for all $n \geq 0$.