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Quiz

S = all living people

h(s) = height of person s

T(k) ≡ Person k is taller than every other person

$$\boxed{\equiv \forall s \in S, \exists k \in S: s \neq k \wedge h(k) > h(s)}$$

INCORRECT

Problems

- k is input variable. DO NOT QUANTIFY INPUT VARIABLES!

STILL INCORRECT

Better: $\boxed{\forall s \in S, s \neq k \wedge h(k) > h(s)}$

"Every person is not person k and every person is shorter than person k"

What about person k?

☆ Test logical connectives: $\wedge, \rightarrow, \vee$

$$\boxed{\forall s \in S, s \neq k \rightarrow h(k) > h(s)}$$

Correct

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2 approaches to inductive step:

1. Start with $P(k)$ = True. Manipulate this expression until you produce $P(k+1)$ = True.

ex:

$$7^k - 1 = 6b \quad \text{multiply by 7 on both sides}$$
$$7^{k+1} - 7 = 6 \cdot 7b \quad \text{add 6 to both sides}$$
$$7^{k+1} - 1 = 6 \cdot 7b + 6$$
$$7^{k+1} - 1 = 6c \qquad P(k+1) \text{ is true}$$

2. Start with part of $P(k+1)$. Plug in $P(k)$. Show $P(k+1)$ = True

$$7^{k+1} - 1 = (7^k - 1) \cdot 7 + 6 = 6b = 6(7 \cdot b + 1)$$

1st part of $P(k+1)$

Because $P(k)$, $7^k - 1 = 6b$ for $b \in \mathbb{Z}$

[Use approach 2 for code proofs]

Don't prove $P(k+1) \rightarrow P(k)$!! Common mistake

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Prove: $2^n - 1 \leq 3^n$ for all integers $n \geq 1$.

[See slides for solution.]

Hint: Start $2^k - 1 \leq 3^k$

$\Downarrow$ Transform

$2^{k+1} - 1 \leq f(k)$

Next show $\Rightarrow$ $f(k) \leq 3^{k+1}$

$\rightarrow$ Transitive property $2^{k+1} - 1 \leq f(k) \leq 3^{k+1}$

$\Downarrow$

$2^{k+1} - 1 \leq 3^{k+1}$

Prove: ReverseString algorithm is correct

For algorithms not always obvious:

- What is "n", the global inductive variable?
- What is base case?

Solution: See slides