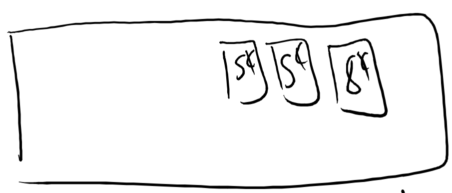
S. KIMMEL

Induction

Recursive algorithm correctness $\longleftrightarrow$ Proof by induction

Example

Suppose you have unlimited 5¢ stamps and 8¢ stamps. What postage values can you create?



18¢ ✓

What about 4¢? No!

What about 28¢? Yes!

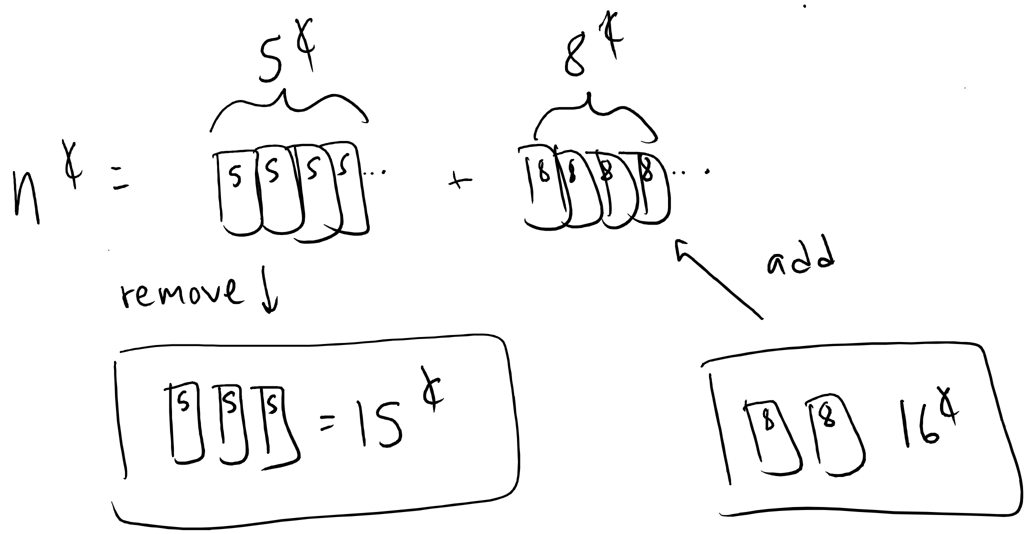
$\boxed{5}\boxed{5}\boxed{5}\boxed{5}\boxed{8}$

What about 85,694¢? ???

S.KIMMEL

Induction: use old solution to get new solution

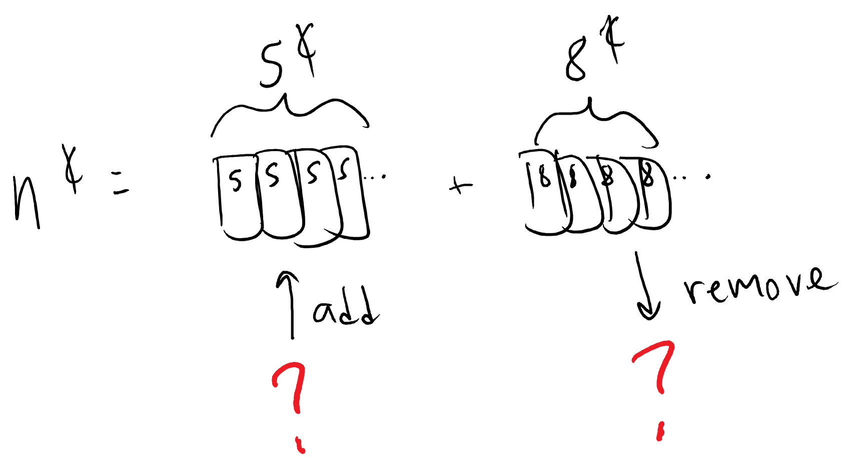
Suppose



$n¢ \to (n+1)¢$

Q:

Suppose

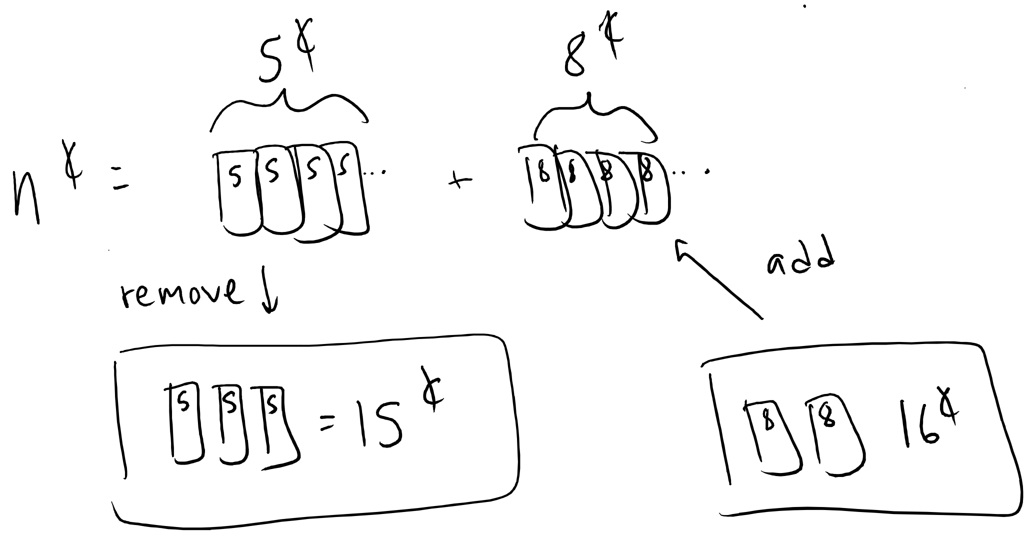


$n¢ \to (n+1)¢$

S.KIMMEL

Induction: use old solution to get new solution

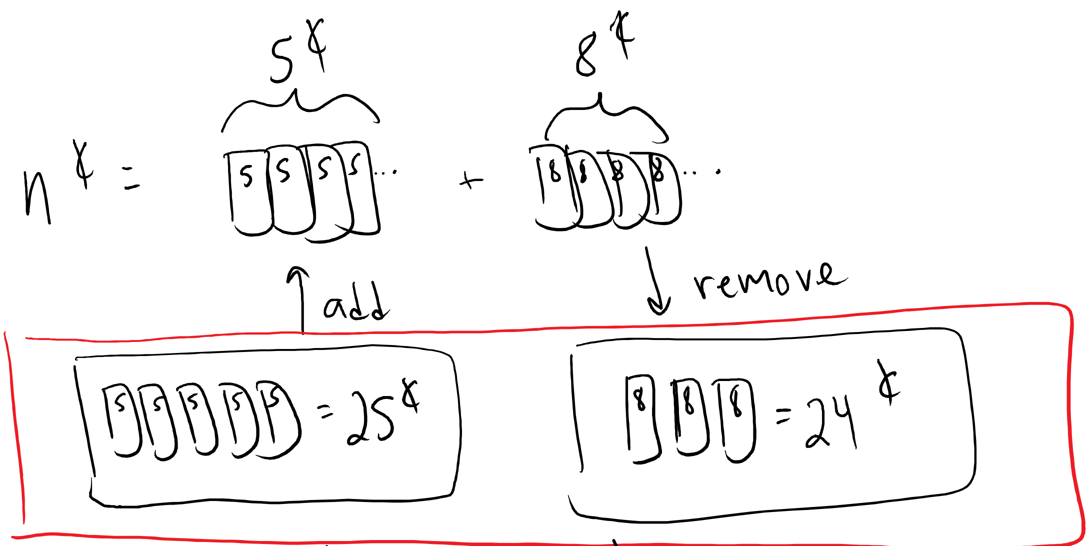
Suppose



$n¢ \to (n+1)¢$

Q:

Suppose



A:

$n¢ \to (n+1)¢$

Consequence: If can create $n¢$ with at least 3 [5] or at least 3 [8], can create $n+1$ ¢

S. KIMMEL

$28¢ = 4 \cdot \boxed{5} + 1 \cdot \boxed{8}$

$29¢ = 1 \cdot \boxed{5} + 3 \cdot \boxed{8}$

$30¢ = 6 \boxed{5}$

$31¢ = 3 \boxed{5} + 2 \cdot \boxed{8}$

$\vdots$

Q: If $85{,}693¢ =$

$5{,}761 \boxed{5} + 7111 \cdot \boxed{8}$

then can create

$85{,}694¢$ as

? $\_\_\_\_\_ \boxed{5} + \_\_\_\_\_ \boxed{8}$

or

$\_\_\_\_\_ \boxed{5} + \_\_\_\_\_ \boxed{8}$

A) 5759 / 7114

B) 5764 / 7108

C) 5766 / 7108

D) 5758 / 7113

S. KIMMEL

$28¢ = 4 \cdot \boxed{5} + 1 \cdot \boxed{8}$

$29¢ = 1 \cdot \boxed{5} + 3 \cdot \boxed{8}$

$30¢ = 6 \boxed{5}$

$31¢ = 3 \boxed{5} + 2 \cdot \boxed{8}$

⋮

Q: If $85{,}693¢ = 5{,}761 \boxed{5} + 7111 \cdot \boxed{8}$

then can create $85{,}694¢$ as

$\underline{5758}\ \boxed{5} + \underline{7113}\ \boxed{8}$

or

$\underline{5766}\ \boxed{5} + \underline{7108}\ \boxed{8}$

Answer:

A) 5759 / 7114

B) 5764 / 7108

C) 5766 / 7108 ⟵

D) 5758 / 7113 ⟵

find first solution, & the rest fall into place

⇓

* Any postage $\geq 28¢$ is possible

Start at $28¢ \to 29¢ \to 30¢ \ldots\ 85{,}693¢ \ldots$

S.KIMMEL

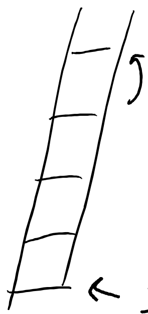
Principle of Induction: solution to smaller problem provides solution to larger problem

Stamps - need to have solution to n to get to n+1

Once you get 28¢ solution, we're good - always at least 3 5¢ or 8¢

Inductive Metaphor

Ladder



2. Show how to move from each rung to next

1. Show how to get on first rung (1st solution)

Shows you can get to all rungs! (1st rung and above.)

S.KIMMEL

Inductive proof recipe:

(Set-Up)

n is always an integer. P(n) is "n^{th} rung of ladder"

Let P(n) be the predicate <u>n¢ of postage can be formed</u> from 5¢ and 8¢ stamps

We will prove, using induction on n, that P(n) is true for all $n \geq$ <u>28</u>.

(Base Case)

Base case: P(<u>28</u>) is true because ______________________

(Inductive Step)

Inductive case: Let $k \geq$ <u>28</u> (base case #). Assume, for induction, that P(k) is true.

That means ______________________

So ______________________

Then ______________________

Plugging in ______________________

P(k) → P(k+1)

Don't go back ward

Thus P(k+1) is true

(Conclusion)

Therefore, by induction, P(n) is true for all $n \geq$ ___.

Q: Put the following sentences in the correct order, and identify and correct any errors.

Then there exists an integer b such that $7^k - 1 = 6b$.

Because b is an integer, $7b + 1$ is an integer, so $P(k + 1)$ is true.

Inductive Step: Let $k \geq 1$ and assume that $P(k)$ is true.

Let $P(n)$ be the predicate $7^n - 1$ is a multiple of 6 for all $n \geq 0$.

Base Case: $P(1)$ is true because $7^1 - 1 = 6$, which is a multiple of 6 since $6 \times 1 = 6$.

We will prove using induction that $P(n)$ is true.

Therefore, by induction on n, $P(n)$ is true for all $n \geq 0$.

Multiplying both sides by 7, we get $7^{k+1} - 1 = 6(7b + 1)$.

S.KIMMEL

NOTE: m is a multiple of 6 if $m = 6 \cdot b$ for an integer b.

Proof that $7^n - 1$ is a multiple of 6 for all $n \geq 0$, with errors corrected:

Let $P(n)$ be the predicate $7^n - 1$ is a multiple of 6. ~~for all $n \geq 0$.~~ We will prove $P(n)$ is true for all $n \geq 0$.

Why?

P is a function that takes in a number and outputs a sentence

- $P(n) \Rightarrow$ " $7^n - 1$ is a multiple of 6 for all $n \geq 0$."

$P(2) \Rightarrow 7^2 - 1$ is a multiple of 6 $\underbrace{\text{for all } 2 \geq 0}_{\text{doesn't make sense}}$

We will prove using induction that $P(n)$ is true.

Base Case: ~~$P(1)$~~ $P(0)$ is true because ~~$7^1 - 1 = 6$~~ $7^0 - 1 = 0$, which is a multiple of 6 since ~~$6 \times 1 = 6$~~. $6 \times 0 = 0$.

Inductive Step: Let ~~$k \geq 1$~~ $k \geq 0$ and assume that $P(k)$ is true.

Then there exists an integer b such that $7^k - 1 = 6b$.

Multiplying both sides by 7, and adding 6 to both sides, we get $7^{k+1} - 1 = 6(7b + 1)$.

Because b is an integer, $7b + 1$ is an integer, so $P(k + 1)$ is true.

Therefore, by induction on n, $P(n)$ is true for all $n \geq 0$.