Announcements

- Please bring laptops on Wednesday and Friday. (Can check out from Davis.)
- Programming Assignment due Tuesday
- Overall reflection due Thursday (see instructions)
- Exam Content: Fully cumulative. Emphasis on most recent topics.
- 2-sided cheat sheet – no other resources

Review Topics

- Recurrence Relations (strings)
- Recurrence relations (pseudocode)
- Indicator Random Variables
- Graph Pseudocode
- Equivalence Relations
- Proof Writing

Indicator Random Variables

Consider an ordered list containing the elements $\{1,2,3,\ldots,n\}$ with no repeats. An inversion is a pair (i,j) where $i < j$ but j precedes i in the list. For example if we consider the ordered list $(3,1,4,2)$ of the elements there are 3 inversions: $\{1,3\}, \{2,3\}, \{2,4\}$.

If an ordering is chosen with equal probability from among all possible orderings, what is the average number of inversions?

1. What is the sample space and random variable of interest?
2. Write rand. variable as weighted sum of indicator rand. variables
3. Use linearity of expectation and property of indicator rand. variables to calculate the expectation.

Indicator Random Variables Solution

1. What is the sample space and key random variable?
   - Sample space is set of $n!$ possible permutations of n elements. Random variable X is the number of inversions in a single permutation
2. Break key random variable into sum of indicator random variables
   - Let X_{ij} take value 1 if there is an inversion between i, j, and 0 else. Then $X = \sum_{ij} X_{ij}$, where the sum is over all unordered pairs of vertices where $i \neq j$.
3. $E[X] = \sum_{ij} E[X_{ij}]$ (using linearity of expectation)

$E[X] = \sum_{ij} \Pr[there\ is\ an\ inversion\ of\ i, j]$. (using property of ind. rand. vars.)

Any two elements are equally likely to be inverted or not! So

$\Pr[there\ is\ an\ inversion\ of\ i, j] = \frac{1}{2}$. And hence

$$E[X] = \sum_{ij} 1/2 = \frac{C(n,2)}{2} = n(n-1)/4$$