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Equivalence Relation

A relation R of a set A is $R \subseteq A \times A$

$$= \{(a,b) : a \in A, b \in A\}$$

An equivalence relation is

- Reflexive $(a,a) \in R \quad \forall \; a \in A$
- Symmetric $(a,b) \in R \iff (b,a) \in R$
- Transitive $(a,b), (b,c) \in R \rightarrow (a,c) \in R$

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A) Equivalence Relation

B) Not reflexive

C) Not symmetric

D) Not transitive

1. $R \subseteq \mathbb{R} \times \mathbb{R}$, $(a,b) \in R \iff a-b \in \mathbb{Z}$

A) Equivalence Relation

1. $a-a=0 \in \mathbb{Z} \quad \forall\, a \in \mathbb{R}$ ✓ Reflexive

2. $a-b \in \mathbb{Z} \iff b-a \in \mathbb{Z}$ ✓ Symmetric

3. $a-b = x \in \mathbb{Z} \quad b-c = y \in \mathbb{Z}$

$x+y \in \mathbb{Z}$

$x+y = (a-b)+(b-c) = a-c \Rightarrow a-c \in \mathbb{Z}$ ✓ Transitive

2. $R \subseteq \mathbb{Z} \times \mathbb{Z}$, $(a,b) \in R \iff a \mid b$

C) NOT Symmetric

$a \mid b \not\iff b \mid a$ 2 divides 4 but 4 does not divide 2

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3. $R \subseteq \mathbb{R} \times \mathbb{R}$, $(a,b) \in R \iff |a-b| < 1$

D) Not transitive

$|1.5 - 1| < 1 \qquad |1 - .3| < 1$

$|1.5 - .3| \not< 1$

4. $R_k \subseteq \mathbb{Z} \times \mathbb{Z} \quad (a,b) \in R_k \iff a \,\%\, k = b \,\%\, k$

$a \equiv b \pmod{k} \qquad a \,\%\, k$ = remainder when a is divided by k

- Reflexive $a \,\%\, k = a \,\%\, k$
- Symmetric $a \,\%\, k = b \,\%\, k$
- Transitive $a = m_1 k + r$ ← (a,b) → $b = m_2 k + r$ ← (b,c) → $c = m_3 k + r$

$a \equiv c \pmod{k}$

Equivalence Class:

$[s] \quad \{a : (a,s) \in R\}$

ex: $[1]_{\text{mod } 5} = \{\ldots -14, -9, -4, 1, 6, 11, 16, \ldots\}$