S. KIMMEL

Today

Review: Recurrence & Random Variables

Announcements: Office Hours 2:35 - 3:30

Recrrence Relation for $T(n)$ ← some function $T: \mathbb{Z}^+ \to \mathbb{Z}^+$

Recurrence Relation: $T(n) = 2T(n-1) + T(n-2)$

Base Case(s): $T(1) = 1 \quad T(2) = 3$

- If have n-1 and n-2, need 2 base cases. Might need to add extra base case after figure out recurrence relation

Why? ⟹

Otherwise run into trouble at $T(3) = 2\underbrace{T(2)}+T(1)$

$2\left(2T(1) + T(0)\right) + T(1)$

0 not allowed input!

To solve: iterate and guess form

S. KIMMEL

Recursive Algorithm:

$$T(n) = A\,T(n-1) + f(n)$$

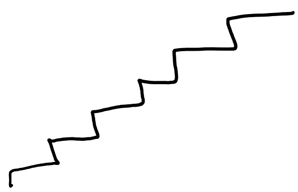
- T: run-time
- n: input size
- A: # of recursive calls
- $(n-1)$: size of recursive call
- $f(n)$: amount of ops done excluding recursive calls, e.g. $O(1)$, $O(n)$, $O(n^2)$

Sometimes $T(n/2)$

Other strategies:

- # of steps required (Towers of Hanoi): use solution to smaller problem to help. Like induction/strong induction

Assume $T(n-1)$ is # of ways to get up $n-1$ stairs



$T(n)$ = # ways to get up n stairs

- # of strings: think about # of choices for final element, and then solve for each case

S. KIMMEL

Let $T(n)$ = # of strings in $\{0,1,2\}^n$ with 2 consecutive 0's or 1's.

Options A: $\_\_\_\_\_\_\_\_ 0$

B: $\_\_\_\_\_\_\_\_ 1$

C: $\_\_\_\_\_\_\_\_ 2$

Using sum rule, need to add possibilities for each option

Let $T_1(n)$ = # 2 consecutive 0's, 1's, ends in 1

$T_0(n)$ = " " 0

$T_2(n)$ = " " 2

C: Need 2 consecutive 0's, 1's in first $n-1$. $\Rightarrow T(n-1)$

A: 3 cases: $\_\_\_\_\_\_\_\_ 00 \Rightarrow 3^{n-2}$

$\underbrace{\_\_\_\_\_\_\_\_ 1}0 \Rightarrow T_1(n-1)$

$\underbrace{\_\_\_\_\_\_\_\_ 2}0 \Rightarrow T_2(n-1)$

+ B: Similarly: $3^{n-2} + T_0(n-1) + T_2(n-1)$

S.KIMMEL

$$\text{Total: } T(n-1) + 2\cdot 3^{n-2} + \underbrace{T_0(n-1) + T_1(n-1) + T_2(n-1)} + T_2(n-1)$$

using sum rule.

$$= T(n-1) + 2\cdot 3^{n-2} + T(n-1) + T_2(n-1)$$

Now $T_2(n-1) = T(n-2)$

$$\underbrace{\_\_\_\_\_\_\_\_\_\_}_{\text{need to have 2 consecutive 0's here}}\ 2$$

$$T(n) = 2\cdot 3^{n-2} + 2T(n-1) + T(n-2)$$

$T(1) = 0$

$T(2) = 2$

Check $T(3) = 2\cdot 3^1 + 2T(2) + T(1)$

$= 6 + 4 = 10$

$$\left.\begin{array}{ll} 00\_ & \times 2 \\ 11\_ & \times 2 \\ \_00 & \times 2 \\ \_11 & \times 2 \\ 000 & \\ 111 & \end{array}\right\} 10$$

S.KIMMEL

Random Variables

36 both sections

Q: On average, how many pairs of people in CS200 same birthday? (If distribution of birthdays is uniform)

- Random variable of interest: # of pairs of people w/ same birthday
- Sample space: all possible birthday combinations : 356^{36}

$X(\text{list of birthdays}) = \#$ of pairs in list with same birthday

- Let X_{ij} be indicator random variable

$$= \begin{cases} 1 & \text{if Person } i \text{ \& Person } j \text{ have same b-day} \\ 0 & \text{else} \end{cases}$$

$$X = \sum_{\substack{i,j \\ \text{possible pairs}}} X_{ij}$$

$$\mathbb{E}[X] = \sum_{i,j} \mathbb{E}[X_{ij}] = \sum_{i,j} \Pr[i,j \text{ have same birthday}]$$

S.KIMMEL

Q: What is $\Pr[i,j]$?

A) $\frac{1}{365}$ B) $\frac{1}{365^2}$

$\Uparrow$

$$\Pr[i,j] = \frac{365}{365^2}$$

$\Leftarrow$ could have same birthday on any day of year

$\Leftarrow$ sample space using product rule

$$\sum_{i,j} \frac{1}{365} = \binom{36}{2}\frac{1}{365} = \frac{36 \cdot 35}{2 \cdot 365} = 1.7$$