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Quantifiers: one way to turn Predicates into Statements

Universal Quantifier: $\forall$ means "for all", "every"

ex: $\forall x, x > 0$ means "for every number x, $x > 0$"

Statements

Existential Quantifier: $\exists$ means "there exists" "there is"

$\exists x : x > 0$ means

"there exists a number x such that $x > 0$"

Q: Is the following true or false? Discuss

$$\forall x, \exists y : x = y$$

A) True

B) False

C) Not enough information to decide

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Depends on the Domains of x, y

$$\forall \underbrace{x \in}_{\text{is an element of}} \underbrace{\mathbb{Z}}_{\text{the set of integers}}, \exists y \in \underbrace{\mathbb{N}}_{\text{the set of natural numbers}} (x = y)$$ False!

the set of integers $\{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$

the set of natural numbers $\{0, 1, 2, 3, \ldots\}$

* Sometimes 0 is not included in natural no's

$x = -1$, an integer, but there is no natural number y s.t. $y = -1$.

$$\underbrace{\forall x \in \mathbb{N}, \exists y \in \mathbb{N} : x = y}_{\text{These types of statements appear often in proofs.}}$$ True!

These types of statements appear often in proofs.

ex:

Let $P(n)$ be the predicate $7^n - 1$ is divisible by 6. We will prove $\boxed{\forall n \in \mathbb{N}, \boxed{P(n)}}$ is true.

statement →

predicate