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Announcements

- I am away Wed./Fri
  - Friday: in-class quiz (Prof Andrews)
  - I will post video lectures to website. You may want to come to class and watch discuss together
- Self-Grade/Reflections: Due by 3 pm Wed to my Mailbox or submitted on Canvas
  - outside MBH 633
- Extra Office Hours on Wed, Thurs, Friday
  Book at Calendly.com/skimmel
  Will use Zoom. I will send Meeting ID
  Go to Middlebury.zoom.us
  Click Join & put in ID
- Extra office hours M, T drop-in

$\forall x \; \exists y$

$\forall x$: nemesis gets to choose from any in domain to try to make false

$\exists y$: You get to choose one specific y based on x from nemesis and prove true

$P(x,y) = x+y=10$

Can get rid of y in predicate using quantifier:

$P(x) = \forall y, \; x+y=10$

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Direct Proof

- Usually used to prove implication like $P \to Q$.

Structure:

Assume P. Explain, explain, ... explain. Therefore Q

"By definition..."

- Also used to prove universal implication: $\forall x\,(P(x) \to Q(x))$

x in some Domain

Structure:

Let x be [any / an arbitrary] element of the domain

Assume $P(x)$

Explain, explain, explain.

Therefore $Q(x)$

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Q: Use a direct proof to show: For all $a, b, c \in \mathbb{Z}$, if $a|b$ and $b|c$ then $a|c$.

($a|b$ means a divides b, that $\exists\, e \in \mathbb{Z}: ae = b$.

Let $a, b, c \in \mathbb{Z}$. Assume $a|b$ and $b|c$. This means $\exists\, e, f \in \mathbb{Z}$ such that $b = ae$ and $c = bf$. Then

$$c = bf = (ae)f = a(ef).$$

But this means $a|c$, since $c = ak$, for $k = ef$, an integer.

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$P \Longleftrightarrow Q$ If and only if

$(P \to Q) \wedge (Q \to P)$ logically equivalent to $P \Leftrightarrow Q$

$\Downarrow$

Structure

For the forward direction, [Proof of $P \to Q$]

For the backward direction [Proof of $Q \to P$]

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Direct Proof - Proof by Contrapositive

- Usually used to prove implication like $P \to Q$.

  Recall: $P \to Q$ is logically equivalent to $\neg Q \to \neg P$

Structure:

We prove the contrapositive

Assume $\neg Q$. Explain, explain, ... explain. Therefore $\neg P$

- Also used to prove universal implication: $\forall x \, (P(x) \to Q(x))$

  x in some Domain

Structure:

Let x be [any / an arbitrary] element of the domain

We prove the contrapositive.

Assume $\neg Q(x)$

Explain, explain, explain.

Therefore $\neg P(x)$

(See example on hw!)

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What if statement is not of the form $P \to Q$?

What if just have P?

Suppose you can show

P	Q	$\neg P$	$\neg Q$	$\neg P \to Q$	$\neg P \to \neg Q$
T	T	F	F	T	T
T	F	F	T	T	T
F	T	T	F	T	F
F	F	T	T	F	T

Proof needs to do two things

① (Direct) $\neg P \to Q$

② (Direct) $\neg P \to \neg Q$

$\therefore P$

Structure: (Prove P)

"For contradiction, assume $\neg P$"

or

"We proceed by contradiction. We assume $\neg P$."

⋮

Therefore, Q

However

⋮

Therefore, $\neg Q$, a contradiction. Thus, P.