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# Big-Omega

**def:** Let $f, g: \mathbb{N} \to \mathbb{R}$. Then $f(x) = \Omega(g(x))$ if there exist positive constants $C$, $K$ such that

$$f(x) > C g(x) \quad \forall x > k$$

**ex:** $x^2 - 5 = \Omega(x^2)$

$$x^2 - 5 > \quad \dots \quad > \quad \dots \quad = C x^2$$

Suppose we can show: $-5 > -c' x^2 \quad \forall x > k'$

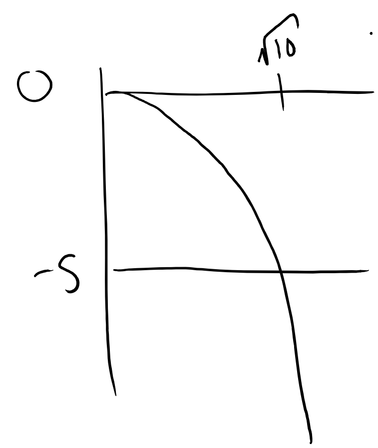
Then $x^2 - 5 > x^2 - c' x^2 = (1 - c') x^2 = C x^2$

→ $c'$ must be less than 1 so that $C > 0$

→ If $c'$ is negative, $-5 > -c' x^2$ is false

→ try $c' = 1/2$

$$-5 > -\tfrac{1}{2} x^2 \quad \forall x > 4$$




$$x^2 - 5 > x^2 - \tfrac{1}{2} x^2 = \tfrac{1}{2} x^2$$

for $x > 4$ $\qquad C = \tfrac{1}{2}, \ k = 4$ ✓

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# Conditional Probability

Let $P(E|F)$ be probability event $E$ occured, if you know event $F$ occured. (Conditional probability of $E$, given $F$)

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

ex: Let $S = \{0,1\}^3 = \{000, 001, 010, \ldots\}$ (length-3 bit strings)

chosen uniformly at random

Let $F$ = first bit is 0

Let $E$ = 2 consecutive zeros

What is $P(E|F)$? What is $P(E)$?

A) $\frac{3}{8}, \frac{3}{8}$ B) $\frac{1}{2}, \frac{3}{8}$ C) $\frac{2}{3}, \frac{1}{2}$ D) $\frac{1}{2}, \frac{2}{3}$

$E = \{000, 001, 100\}$ $P(E) = 3/8$

$E \cap F = \{000, 001\}$ $P(E \cap F) = 2/8$

$F = \{000, 001, 010, 011\}$ $P(F) = \frac{4}{8} \Rightarrow P(E|F) = \frac{1}{2}$

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# Independent Events

$E, F \subseteq S$ are independent $\Leftrightarrow$

$$P(E) = P(E|F)$$

(Probability of E occurring is doesn't depend on whether F occured.)

Thm: If $E, F$ are independent,

$$P(E \cap F) = P(E) \cdot P(F)$$

Pf:

$$\frac{P(E \cap F)}{P(F)} = P(E|F) = P(E)$$

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Q: Suppose you have a di where $P(6)=\frac{1}{2}$, $P(1)=P(2)=\dots=P(5)=\frac{1}{10}$. What is the probability of getting two 6's out of 4 rolls? (Order matters.)

A. $S=\{1,2,3,4,5,6\}^4$

$E=\{i : i \text{ contains } 2\ 6\text{'s}\}$

$$\Pr(E)=\sum_{i\in E}\Pr(i)$$

Because each roll is independent:

$$\Pr(6\ 6\ 1\ 2)=\Pr(6)\cdot\Pr(6)\cdot\Pr(1)\cdot\Pr(2)$$

$$=\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{10}\cdot\frac{1}{10}=\frac{1}{400}$$

If swich order, $\Pr(i)$ is still same!

$$\Rightarrow \Pr(E)=\sum_{i\in E}\frac{1}{400}=\frac{|E|}{400}=\frac{150}{400}$$

Using product rule: $|E|=\underbrace{\binom{4}{2}}_{\text{places where 6 can be}}\cdot\underbrace{5}_{\text{choice for first non 6}}\cdot\underbrace{5}_{\text{choice for second non 6}}=150$