

Recall Sum/Product/Subtraction

Q: There are 10 singles left in Coffin and you and 2 friends want to pick 3 of them.

How many ways could you choose rooms.

- A) 30 B) 300 C) 720 D) 1000

Answer: using product rule $10 \cdot 9 \cdot 8 = 720$

New function:

$P: \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$. $P(n, k)$ is the number of k -permutations of n elements.

e.g. $P(10, 3) = 720$.

Q:

What is a permutation?

- An ordering of a set of elements

What is a k-permutation?

- An ordering of a set of k elements

What is a k-permutation of n elements

- An ordering of a set of k elements where those k are chosen from n elements

What is a formula for $P(n, k)$? Using product rule:

$$n \cdot (n-1) \cdot (n-2) \cdots (n-k+1)$$

$$\text{or } \prod_{i=n-k+1}^n i \quad \text{or } \prod_{j=0}^{k-1} (n-j)$$

How many permutations are there of n elements?

$$- n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdots \cdot 1 = n!$$

Another way to write $P(n, k)$:

$$10 \cdot 9 \cdot 8 \left(\frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \right) = \frac{10!}{7!}$$

Where $n! = n \cdot (n-1) \cdot (n-2) \cdots \cdot 1 = \prod_{i=1}^n i$

so $P(n, k) = \frac{n!}{(n-k)!}$

Q: There are 10 singles left in Coffin and you and 2 friends want to pick 3 of them.

Suppose you just want to pick 3 rooms now, and you'll figure out who will stay where later.

How many ways could you pick 3 rooms?

- A) 30 B) 120 C) 240 D) 360

We know 720 ways if care about order.

So : $(2, 3, 5), (2, 5, 3), (3, 2, 5), (3, 5, 2)$
 $(5, 2, 3), (5, 3, 2)$
 $\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ \text{My} & \text{Friend} & \text{Friend} \\ \text{pick} & \text{1} & \text{2} \\ & \text{pick} & \text{pick} \end{array}$

But if don't care about order, these are all the same. $\{2, 3, 5\}$

\Rightarrow Over counting by a factor of 6 for each set!

$$720/6 = 120$$


Function

$C(n, r) = \binom{n}{r}$ = "n choose r" is the number of sets of r elements chosen from a set of n elements.

Fact: $\binom{n}{r} = \frac{n!}{r! \cdot (n-r)!}$

Pf: $P(n, r) = \binom{n}{r} \cdot P(r, r)$

Why?

$$\Rightarrow \binom{n}{r} = \frac{P(n, r)}{P(r, r)} = \frac{n!}{(n-r)! \cdot \left(\frac{r!}{1!}\right)} = \frac{n!}{(n-r)! \cdot r!}$$


The number of ways we can order r things chosen from among n things is equal to the number of subsets of r things, times the ways we can order each subset.

Q: IF 8 people from a basketball team show up to a game, how many ways are there to form a 5 person team?

- A) 40 B) 56 C) 60 D) 112

$$\frac{8!}{5!3!} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 8 \cdot 7 = 56$$