Problem Set 0 - Sample Solution

by Alice Exception

1. [**11 points**] Prove $7^n - 1$ is divisible by 6 for all integers $n \geq 0$.

Solution

Let $P(n)$ be the predicate $7^n - 1$ is divisible by 6 for all integers $n \geq 0$. We will prove using induction that $P(n)$ is true for all $n \geq 0$.

Base case: $P(1)$ is true because $7^1 - 1 = 6$, and 6 is divisible by 6.

Inductive step: Let $k \geq 0$. We assume $P(k)$ is true, and we will prove $P(k+1)$ is true. Since $P(k)$ is true, there is some integer m such that $7^n - 1 = 6m$. Then

$$7^{k+1} - 1 = (7^k - 1) \times 7 + 6 = 6m \times 7 + 6 = 6(7m + 1) \tag{1}$$

Thus $P(k+1)$ is true.

Therefore, by induction, $P(n)$ is true for all $n \geq 0$.