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Now we can get back to proofs!

Truth Table Proofs

Prove the following statement is true:

If you eat a raw egg everyday ← P
you will win the lottery, ← Q
or
if you win the lottery you ← R
will lose your job.

1. Identify atomic statements and label
2. Write statement symbollically

$$(P \rightarrow Q) \vee (Q \rightarrow R)$$

3. Create truth table

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start with atomic

next build up to full

P	Q	R	$P \to Q$	$Q \to R$	$(P \to Q) \vee (Q \to R)$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

all possible combinations

Proof: Let P be the statement ....

" Q "

" R "

Then looking at the truth table, we see $(P \to Q) \vee (Q \to R)$ is true for any truth value of P, Q, and R.

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Truth tables are only useful when you don't have too many atomic statements $\left(\#\text{ of rows} \sim 2^{(\#\text{ of atomic statements})}\right)$

Better if can avoid

Q. Explain why $(P \to Q) \vee (Q \to R)$ is true without a truth table

- Q is true or false. If it is true, $P \to Q$ is true. If it is false, $Q \to R$ is true. Either way, at least one of $P \to Q$, $Q \to R$ is true, so the whole statement is true.

Useful tool: Logical Equivalences

ex: $P \to Q$ is logically equivalent to $\neg P \vee Q$

P	Q	$P \to Q$	$\neg P$	$\neg P \vee Q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Q: Based on this example, what is a reasonable definition of logical equivalence?

A:
Statements R and S are logically equivalent if they have the same truth value under any assignment of truth values to their atomic parts, i.e. if $R \leftrightarrow S$.

In any statement, can replace $P \to Q$ with $\neg P \vee Q$!

Prove (again) that $(P \to Q) \vee (Q \to R)$ is true using a logical equivalence

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Proof: We will prove $(P \to Q) \vee (Q \to R)$ is true for any statements P, Q and R. Using the logical equivalence of $P \to Q$ and $\neg P \vee Q$, we have that $(P \to Q) \vee (Q \to R)$ is logically equivalent to $(\neg P \vee Q) \vee (\neg Q \vee R)$. This expression is true if any of the statements $P, Q, R, \neg Q$ are true. But either Q or $\neg Q$ is true so the statement is true.