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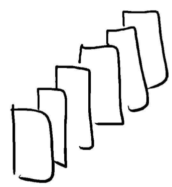
Quiz! (Work on: Prove $2^n + 1 \le 3^n$ for all $n \ge 1$)

Announcements: • HW submitted through CANVAS (pdf)
↑
• Scanners
• Type

• No late accepted
• Submit early to test
• New students
• Lots of practice problems in readings

Dominos as metaphor for induction:



2: Show each domino falling will knock next one over

1: Show how to knock down first domino

Parts of Inductive Proof

1. Set-up (state problem, approach)
2. Base-case: (1st solution)
3. Inductive step/case: ($k^{th} \to (k+1)^{th}$ solution)
4. Conclusion (put a bow on it!)

Purpose

(Tell them what you're going to say, say it, tell them what you said)

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Inductive Step Strategy

$\swarrow$ P(k) is true

1. Phrase inductive assumption using as much math as possible

   e.g. Instead of: 7^k-1 is divisible by 6,
   Better: $7^k-1=6m$ for an integer m

2. Write "k side" of P(k+1) as math and use series of transforms and substitution from Step 1 to reach other side

$$
\begin{aligned}
7^{k+1}-1 &= \underline{7^k\cdot 7-1} && \text{transform}\\
&= \underline{(7^k-1)\cdot 7+7-1} && \text{transform}\\
&= \underline{(7^k-1)\cdot 7+6} && \boxed{\text{Substitute equation from Step 1}}\\
&= \underline{6m\cdot 7+6} && \text{transform}\\
&= \underline{6(m\cdot 7+1)} && \text{transform}\\
&= \underline{6m'}
\end{aligned}
$$

Need words here "Plugging in from the inductive assumption, we get

Now $7^{k+1}-1$ is divisible by 6.

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Prove: The sum of the first n odd numbers is n^2 for $n \geq 1$.

Q: Which of the following represents the sum of the first n odd numbers? (If $n \geq 3$)

A) $1 + 3 + \cdots + (2n-1)$

B) $1 + 3 + \cdots + (2n+1)$

C) $1 + 3 + \cdots + (2n-3)$

D) $1 + 3 + \cdots + (2n+3)$

n	sum
1	1
2	1+3
3	1+3+5
4	1+3+5+7

Odd numbers: 1, 3, 5, 7, 9, 11, 13

Prime numbers: only divisible by 1 andIt

1, 2, 3, 5, 7, 11, 13

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Proof:

- Let $P(n)$ be the predicate the sum of the first n odd numbers is n^2. We will prove using induction on n that $P(n)$ is true for all $n \geq 1$.

- Base case: The first odd number is 1, and $1 = 1^2$, so $P(1)$ is true.
- Inductive case: Let $k \geq 1$. Assume for induction that $P(k)$ is true. We will prove $P(k+1)$ is true. Using the inductive assumption

$$1 + 3 + \cdots + (2k-1) = k^2$$

Then the sum of the first $k+1$ odd integers is

$$1 + 3 + \cdots + 2k-1 + 2(k+1)-1$$

Plugging in from inductive assumption, this is

$$k^2 + 2(k+1) - 1 = k^2 + 2k + 1 = (k+1)^2$$

Thus, $P(k+1)$ is true.

- Therefore, by induction, $P(n)$ is true for all $n \geq 1$.