

# Counting

Q: Suppose you have a combination lock. It has 3 dials, each with the numbers  $1, 2, \dots, 20$ . If it takes you one second to test each combination, how long will it take you to test all combinations?

A) 60 sec.    B) 400 sec.    C) 6,840 sec.    D) 8000 sec.

$\sim 2$  hours

This is why passwords should be long! Otherwise can exhaustively check all options. With 5 dials  $\rightarrow \sim 37$  days

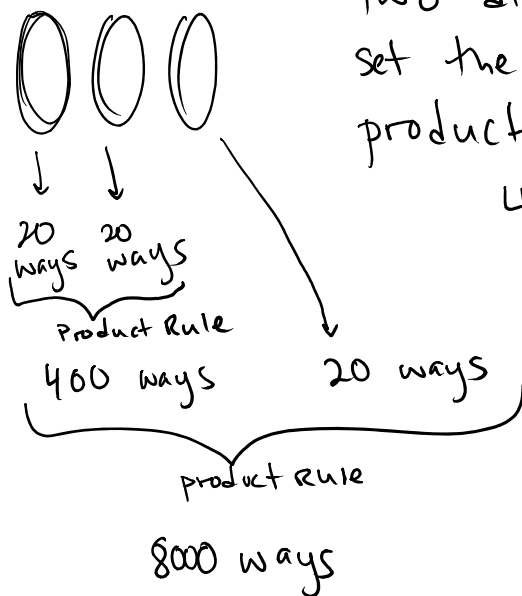
Probably figured out using intuition. Here is a rule to help you solve these types of problems

Product Rule: Suppose a procedure can be broken down into two tasks, and there are  $n_1$  ways to do the first task and  $n_2$  ways to do the second. Then there are  $n_1 \times n_2$  ways to do the procedure

Q: Explain why answer to previous question is 8000 using product rule.

A: Let's first consider the procedure of setting the first two dials. There are 20 ways to set the first dial, and 20 ways to set the second dial. So by the product rule, there are  $20 \times 20 = 400$  ways to set the first two dials. Now consider the procedure of setting all dials. There are 400 ways to set the first two dials and 20 ways to set the last dial, so by the product rule, there are

$400 \times 20 = 8000$  ways to set all dials.



## SUM RULE

If you have a task that can be done in one of  $n_1$  ways or one of  $n_2$  ways, where the  $n_1$  ways are all different from the  $n_2$  ways, then there are  $n_1 + n_2$  ways of completing the task.

## Combining Product & Sum Rule

Q: Suppose you and your best friend are picking into a coffee house. There are 20 singles left and 3 doubles. If you both choose singles or both share a double, how many options of room choices do you have.

- A) 23    B) 383    | C) 403    D) 1200

If you both pick singles, there are 2 tasks:

1<sup>st</sup> person  
chooses



20 ways

2<sup>nd</sup> person  
chooses



19 ways

= 380 ways

There are two options: singles or doubles



380 ways



3 ways

= 383

\* Doesn't matter who chooses first

I choose

Rm 10

| You choose

Rm 4

or

I choose

Rm 4

| You choose

Rm 10

## Subtraction Rule

Let  $A_1, A_2$  be sets

$$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$$

Similarly,

If you can do a task  $n_1$  ways or  $n_2$  ways,

then the total number of ways to do a task is  $n_1 + n_2$  minus the number of ways common to the two approaches.