Strategy

1. A & B start with $|\psi\rangle_{A_1} |\beta_{00}\rangle_{A_2 B}$ (1 ebit)

2. Alice measures $A_1$ and $A_2$ using Bell Basis
   (this destroys entanglement)

3. Alice sends outcome of measurement to Bob (2 cbits)

4. Bob applies a unitary to his system $B$ based on Alice’s cbits

What state does Bob end up with in each case?

Which unitary should Bob apply for each outcome?
Alice measures qubits $A_1$ & $A_2$ in Bell basis (recall strategy from composite systems)

\[
|\psi\rangle_{A_1} |\beta_{00}\rangle_{A_2B} = (a|0\rangle + b|1\rangle) \otimes \frac{1}{\sqrt{2}} (|100\rangle + |111\rangle) = \frac{1}{\sqrt{2}} (a|000\rangle + a|011\rangle + b|100\rangle + b|111\rangle)_{A_1A_2B}
\]

\[
|\psi\rangle |\beta_{00}\rangle = \\
+ |\beta_{00} \rangle \langle \beta_{00} |_{A_1A_2} \otimes I_B |\psi\rangle |\beta_{00}\rangle \quad = \quad \frac{1}{2} |\beta_{00}\rangle \langle a|0\rangle + b|1\rangle \\
+ \frac{1}{2} |\beta_{01}\rangle \langle a|1\rangle + b|0\rangle \\
+ \frac{1}{2} |\beta_{01}\rangle \langle a|1\rangle - b|0\rangle \\
+ \frac{1}{2} |\beta_{11}\rangle \langle a|1\rangle - b|0\rangle
\]

**Example of calculation (line 4)**

\[
|\beta_{00} \rangle \langle \beta_{00} |_{A_1A_2} \otimes I_B |\psi\rangle |\beta_{00}\rangle = \frac{1}{\sqrt{2}} \left[ \langle \beta_{00} | 0 \rangle a |\beta_{00}\rangle |0\rangle + \langle \beta_{00} | 1 \rangle a |\beta_{00}\rangle |1\rangle \\
+ \langle \beta_{00} | 1 \rangle b |\beta_{00}\rangle |0\rangle + \langle \beta_{00} | 0 \rangle b |\beta_{00}\rangle |1\rangle \right]
\]

\[
= \frac{1}{\sqrt{2}} \left[ \frac{a}{\sqrt{2}} |\beta_{00}\rangle |0\rangle + \frac{b}{\sqrt{2}} |\beta_{00}\rangle |1\rangle \right] = \frac{1}{2} |\beta_{00}\rangle \langle a|0\rangle + b|1\rangle \\
\]

- All 4 outcomes occur with equal probability $\left(\frac{1}{4}\right)$
<table>
<thead>
<tr>
<th>Alice's Outcome</th>
<th>Bob's State B</th>
</tr>
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<tbody>
<tr>
<td>(</td>
<td>\psi^{0}\rangle)</td>
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<td></td>
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<tr>
<td>(</td>
<td>\psi^{1}\rangle)</td>
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<tr>
<td>(</td>
<td>\psi^{2}\rangle)</td>
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</table>

2. Alice uses 2 qubits to tell Bob her outcome.

3. Based on this info, Bob applies \(I, X, Z, (ZX)^{n}\) to \(B\), recovers the state \(|\psi\rangle = a |0\rangle + b |1\rangle\).

**Big Picture:**

Alice's qubit, which was in state \(|\psi\rangle = a |0\rangle + b |1\rangle\), is now the state of Bob's qubit. She never had to communicate \(a, b\). It just shows up in Bob's possession. Pretty crazy!
Classical Models of Computation (that helped inspire quantum model)

- Circuit (usual classical model)
- Reversible (b/c unitaries are reversible)
- Probabilistic (b/c measurements are probabilistic)

Circuit - way of describing functions on bits

\[ \begin{align*}
\text{bit: } x \in \{0,1\} & \quad \text{time } \rightarrow \\
\text{implement the function} & \\
\text{set of all } 3\text{-bit strings} & \text{eg. } \{0,1\}^3 = \{00,01,10,11\} \\
\end{align*} \]

Gate ex:

\[ \begin{align*}
x_1 & \quad x_2 \quad x \leftarrow x \quad x_1 \quad x_2 \quad x \\
\text{AND} & \quad \text{NOT} & \quad \text{OR} & \\
\text{universal} & \quad \text{universal} & & \\
\end{align*} \]

Gate set is universal, can compute any \( f : \{0,1\}^n \rightarrow \{0,1\}^m \)

Generalized Boolean function

Circuits provide way of determining how difficult it is to compute a function.

Measures of circuit complexity:

- Gate count = # gates used
- Depth = # time steps (depends on parallelization)
- Width/Size = max # wires present at a time

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Ex: find circuit that implements function \( f: \{0,1\}^3 \rightarrow \{0,1\} \); \( f(x,y,z) = \begin{cases} 1 & x_1 \oplus x_2 \oplus x_3 = x_4 = 0 \\ 0 & \text{otherwise} \end{cases} \)

- What is gate count, depth, width?