CS333 - Final Review

1. Consider the following two-qubit state:

$$\frac{1}{\sqrt{3}}\left(|0\rangle|+\rangle + |-\rangle|1\rangle\right) \tag{1}$$

   (a) **[3 points]** If you measure the first qubit of $|\psi\rangle$ using the basis $\{|0\rangle, |1\rangle\}$, what is the probability of getting outcome $|0\rangle$?

   (b) **[3 points]** If you get outcome $|0\rangle$ on the first qubit, what state will the second qubit collapse to?

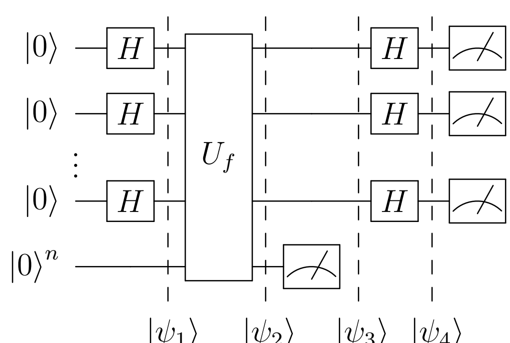
   (c) **[3 points]** If you measure the first qubit of $|\psi\rangle$ using the basis $\{|+\rangle, |-\rangle\}$, what is the probability of getting outcome $|0\rangle$?

   (d) **[3 points]** If you get outcome $|+\rangle$ on the first qubit, what state will the second qubit collapse to?

2. Suppose you have a function $f : \{0,1\}^n \rightarrow \{0,1\}^n$, where $f(x) = f(y)$ if and only if $x = y \oplus s$ for some $s \in \{0,1\}^n$. (Here $\oplus$ means addition mod 2 for each bit of the string.) Otherwise, there is no structure in terms of which input gets assigned to which output.

   (a) What is the classical query complexity of determining s?

   (b) Suppose you have a unitary that acts as $U_f|x\rangle|0\rangle = |x\rangle|f(x)\rangle$. (For this algorithm, it doesn't matter how U_f acts on other standard basis states.) If we run the following algorithm:



(2)

   what are the states at each point? Hints:

- $(a \oplus b) \cdot c \equiv a \cdot c + b \cdot c \mod 2$.
- There should be two standard basis states in superposition after the measurement of the second register.
- Recall how $H^{\otimes n}$ transforms standard basis states.
- Use linearity, obv!

(c) Here is the classical post-processing fact. If you can learn randomly chosen z such that $z \cdot s = 0$, then using $O(n)$ such z, can figure out s. Use this fact to determine the quantum query complexity of learning s.