Teleportation

Resources

|ψ⟩ = 1/\sqrt{12} (|00⟩ + |11⟩)

"cbit" = 1 bit classical info
"qbit" = 1 qubit
"ebit" = 2 qubits in entangled state

Question: If Alice wants to send something to Bob, what resources does she need?

- How many qubits need to be sent to communicate 1 cbit?
  A) 0  B) 1  C) 2  D) ∞

- How many ebits are required to communicate 1 cbit?
  A) 0  B) 1  C) 2  D) ∞ is not enough

- How many cbits are needed to communicate 1 qbit?
  A) 1  B) 2  C) log in precision  D) ∞ is not enough
Teleportation

Resources

"e bit gets used up by being measured or being sent to Bob"

\[ |\beta_{01}\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) \]

"c bit" = 1 bit classical info

"q bit" = 1 qubit

"e bit" = 2 qubits in entangled state

Question: If Alice wants to send something to Bob, what resources does she need?

<table>
<thead>
<tr>
<th>Task</th>
<th>Resources Required</th>
</tr>
</thead>
<tbody>
<tr>
<td>Send 1 cbit</td>
<td>• 1 cbit</td>
</tr>
<tr>
<td></td>
<td>or</td>
</tr>
<tr>
<td></td>
<td>• 1 q bit</td>
</tr>
<tr>
<td></td>
<td>(A sends</td>
</tr>
<tr>
<td></td>
<td>or</td>
</tr>
<tr>
<td></td>
<td>• 1 e bit</td>
</tr>
<tr>
<td></td>
<td>can't communicate with e bit</td>
</tr>
</tbody>
</table>

| Send 2 cbits| • 2 cbit |
|            | • 2 q bit |
|            | • 1 e bit + 1 q bit |

"Superdense coding" don't have time to discuss. Can read about
<table>
<thead>
<tr>
<th>Task</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Send 1 gbtt</td>
<td>• 1 cbits</td>
</tr>
<tr>
<td></td>
<td>• 1 qbits</td>
</tr>
<tr>
<td></td>
<td>• 1 ebit + 2 cbits</td>
</tr>
</tbody>
</table>

**Strategy**

1. A & B start with $|\psi\rangle_{A_1} \rightarrow_{A_2 B}$ (1 ebit)
2. Alice measures $A_1$ and $A_2$ (this destroys entanglement)
3. Alice sends outcome of measurement to Bob (2 cbits)
4. Bob applies a unitary to his system $B$ based on Alice's cbits

New skill: What happens when only part of system is measured?

Recall: If both measured, effective measurement is $M_A \otimes M_B$
Partial Measurement

Let $|\Psi\rangle_{AB}$ be a state on systems $A$ ($N_A$-dim) and $B$ ($N_B$-dim).

\[ \begin{array}{cc}
\text{A} & \text{B} \\
\text{Alice's System} & \circ & \circ \text{Bob's System}
\end{array} \]

- Alice measures with $M_A = \{ |\phi_1\rangle, ..., |\phi_{N_A}\rangle \}$.
- Bob does not measure → if Alice gets outcome $|\phi_i\rangle$, what happens to Bob's state?

1. It is always possible to write (uniquely)
\[ |\Psi_{AB}\rangle = \sum_{i=0}^{N_A} a_i |\phi_i\rangle_A |X_i\rangle_B \]

\[ \sum_{i=0}^{N_A} |a_i|^2 = 1 \]

2. Then Alice gets outcome $|\phi_i\rangle$ with probability $|a_i|^2$, and Bob's system collapses to:
\[ |\phi_i\rangle_A |X_i\rangle_B \]
How to change basis

Orthornomal basis \{ |\psi_i\rangle \}

\[ \mathbb{I} = \sum_i |\psi_i \rangle \langle \psi_i | \]

Identity matrix: \[ (a\ b) \rightarrow (aa^* \ b^*) \]

Ketbra \Rightarrow matrix

Write \(|\phi\rangle\) using \{ |\psi_i\rangle \} basis:

\[ |\phi\rangle = \mathbb{I} |\phi\rangle = \sum_i |\psi_i \rangle \langle \psi_i | |\phi\rangle = \sum_i (\langle \psi_i | \phi \rangle) |\psi_i\rangle \]
\[ |\psi\rangle = \frac{1}{\sqrt{2}} (|00\rangle_{A_b} + |11\rangle_{A_b}) \]

Measure \( A \) in \( \{|+\rangle, |-\rangle\} \)
- What outcomes occur with what probability?
- What happens to \( B \) system?

1. Put system \( A \) into \( \{|+\rangle, |-\rangle\} \) basis:

\[ I_{A_b} = I_A \otimes I_B = (|+\rangle + |-\rangle) \otimes I_B \]

\[ I_{A_b} |\psi\rangle = (|+\rangle + |-\rangle) \otimes I_B \left( \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)_{A_b} \right) \]

\[ = \frac{1}{\sqrt{2}} \left( (|+\rangle_{A_b} |0\rangle_{B} + |-\rangle_{A_b} |1\rangle_{B}) + (|+\rangle_{A_b} |0\rangle_{B} + |-\rangle_{A_b} |1\rangle_{B}) \right) \]

\[ = \frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{2}} |+\rangle |0\rangle + \frac{1}{\sqrt{2}} |+\rangle |1\rangle + \frac{1}{\sqrt{2}} |-\rangle |0\rangle + \frac{1}{\sqrt{2}} |-\rangle |1\rangle \right) \]

\[ = \frac{1}{2} \left( |+\rangle (|0\rangle + |1\rangle) + |-\rangle (|0\rangle - |1\rangle) \right) \]

\[ \text{not normalized} \]

\[ = \frac{1}{\sqrt{2}} \left( |+\rangle \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) + |-\rangle \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) \right) \]

\[ = \frac{1}{\sqrt{2}} \left( |+\rangle |+\rangle + |-\rangle |-\rangle \right) \]

\[ |+\rangle: \Pr \left( \frac{1}{2} \right) \quad B \rightarrow |+\rangle \]

\[ |-\rangle: \Pr \left( \frac{1}{2} \right) \quad B \rightarrow |-\rangle \]