Runtime

Initialize each $i \in \Sigma$ as tree

While (> 1 tree to be merged)
  - Find 2 trees with smallest probability
  - Merge into new tree with new probability

$=$ sum of old probabilities

Q. What is the runtime?

A) $O(n)$  B) $O(n \log n)$  C) $O(n^2)$  D) $O(n^2 \log n)$
Runtime

Initialize each $i \in \Sigma$ as tree \(\leftarrow O(n)\)

While ($\geq 1$ tree to be merged) \(\leftarrow O(n)\) reps
  * Find 2 trees with smallest probability \(\leftarrow\) Extract twice $O(\log n)$
  * Merge into new tree with new probability \(\rightarrow O(1)\)
  * Reinsert into heap : $O(\log(n))$

Q. What is the runtime?

A) $O(n)$ B) $O(n \log n)$ C) $O(n^2)$ D) $O(n^2 \log n)$

\[\uparrow\text{Keep finding min over in over} \\]
\[\text{minJa changing data structure} \]

Use min-heap
  * Initialize $n$ elements in $O(n)$
  * Extract min elt in $O(\log n)$
  * Insert a new elt in $O(\log n)$

Using different data structure, can acheive $O(n \log \log n)$

van Emde Boas tree
Probability Questions

ex: n coin tosses, \( \frac{1}{4} \) prob of heads. How many heads on average?

1. Sample space
   \[ \{H,T\}^n \] = all strings of length \( n \) consisting of heads and tails

2. What is key random variable (look at question)
   \( X = \# \) of heads

3. Write as sum of indicator random variables
   \[ X = \sum X_i \]
   \[ X_i = \begin{cases} 1 & \text{if } i\text{th is head} \\ 0 & \text{if tails} \end{cases} \]

\[ (H\ T\ T\ H) \]
\[ X_1 = 1 \]
\[ X_2 = 0 \]
\[ X_3 = 0 \]
\[ X_4 = 1 \]

\[ \sum X_i = 2 = \# \text{ of heads} \]
3. \[ E[X] = E[\sum X_i] = \sum_i E[X_i] \]

* For indicator random variables,
\[ E[X_i] = P_r(\text{Event associated with 1-value of } X_i) \]
\[ = P_r(\text{Get heads on flip } i) \]
\[ = \frac{1}{4} \]

\[ E[X] = \sum_{i=1}^{n} \frac{1}{4} = \frac{n}{4} \]

Each group prepare to explain \( \frac{2}{|j-i|+1} \) ... random

(Each person must say something.)
MWIS on a binary tree

Weights

\[
\begin{array}{cccc}
10 & 5 & 1 & 9 \\
1 & 2 & 3 & 4 \\
\end{array}
\]

1. Options for optimal soln:
   (i) 1 is in MWIS
   (ii) 1 is not in MWIS

2. Form of solution as recurrence relation
   (i) \( S(T_1) = S(T_4) + S(T_5) + S(T_6) + S(T_7) + 1 \)
   (ii) \( S(T_1) = S(T_2) + S(T_3) \)

Can be a function of multiple variables:

- Knapsack: Capacity, items
- Aluminum sheets: Dimension A, Dim B
- DC/Washington: Week, current location
  only 2 options

\[
\begin{array}{c}
\uparrow \quad y \\
\times \quad 1 \\
\downarrow \quad a \quad n
\end{array}
\]