1. Probability Questions

(a) If you have a coin that has 1/4 probability of heads and 3/4 probability of tails, what is the sample space if you flip it \( n \) times? What is the expected number of heads? (Use indicator random variables).

(b) What happens to the randomized search with replacement algorithm from the homework, if there are two elements with value \( x \) in the array?

(c) Explain why the probability of comparing \( z_i \) and \( z_j \) in Randomized QuickSort is \( 2/(|j - i| + 1) \). (We discussed this in class - please explain in your own words.)

2. Suppose you have a graph \( T \) that is a binary tree, with weights on each vertex. Let \( T_v \) be the subtree with root \( v \). Let \( S(T_v) \) be the max-weight-independent set on \( T_v \) and let \( W(T_v) \) be the weight of the max-weight independent set on \( T_v \). Go through the steps of creating a dynamic programming algorithm.

(a) What are the options for the optimal solution.

(b) For each option, what is the form of the optimal solution in terms of the optimal solution of subproblems.

(c) Use this analysis to create a recurrence relation for the maximum value of the objective function.

(d) Write pseudocode to fill in an array with values of the objective function.