Learning Goals

• Describe MWIS & Scheduling problems
• Design a recurrence relation for MWIS
• Understand process of designing and testing a greedy algorithm.

Sample Syllabus Quiz Question:
Q: Which of the following problem set parts are graded for correctness?
  A. Rough Draft.
  B. Main PSet Submission
  C. Self Grade
  D. None of them - All graded on effort, although you will get the most out of self-grade if you try for accuracy.

Assorted Stuff
• Rough Draft (due today) — anything that shows some effort
• PSet Due Sunday @ 9 pm (tutoring 7-9 Sunday, Wed)
• Syllabus Quiz Monday
• Scheduling Changes → will discuss next week
Max-Weight Independent Set Problem (MWISP)

Input: Graph $(V, E)$ and weight function $w: V \rightarrow \mathbb{Z}^+$

Output: $S \subseteq V$ s.t. if $(v_i, v_j) \in E$, \( v_i, v_j \) can't both be in $S$.

\[ w(S) = \sum_{v \in V} w(v) \text{ is maximized} \quad \text{max weight} \]

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This set is Max Weight Ind. Set (MWIS)
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Applications
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- WiFi transmitters/cell towers
  - Tower $i$ has $n(i)$ packets to broadcast
  - If two towers are within 2 miles and broadcast at same time, causes interference

Dynamic Programming

Recurrence Relation

Algorithm
Q: How to use MWIS to determine which towers should transmit? What is \( V, E \) ? What is \( w : V \rightarrow \mathbb{R} \)?

MWISP on Path Graph

Q: What is MWIS of

\[
\begin{align*}
&v_1 \quad v_2 \quad v_3 \quad v_4 \quad \ldots \quad v_k \\
&5 \quad 7 \quad 6 \quad 1 \quad \ldots
\end{align*}
\]

\[
\begin{align*}
&v_1 \quad v_2 \quad v_3 \quad v_4 \\
&1 \quad 6 \quad 7 \quad 5
\end{align*}
\]

A) 11    B) 13    C) 19    D) None exists
Q: How to use MWIS to determine which towers should transmit? What is \( V, E \)? What is \( w: V \rightarrow \mathbb{R}^+ \)?

- Vertices are towers
- Put an edge b/t any two vertices that are 2 miles or less apart

\[ w(i) = n(i) \]

MWISP on Path Graph

Q: What is MWIS of

\[
\begin{align*}
V_1 & \quad V_2 & \quad V_3 & \quad V_4 & \quad V_5 \\
1 & \quad 6 & \quad 7 & \quad 5 & \\
\end{align*}
\]

A) 11  
B) 13  
C) 19  
D) None exists

\[
\begin{align*}
\text{Best} & \\
V_1 & \quad V_2 & \quad V_3 & \quad V_4 \\
1 & \quad 6 & \quad 7 & \quad 5 \\
\end{align*}
\]

\[
\begin{align*}
w(\{V_1, V_3\}) & \rightarrow 8 \\
w(\{V_2, V_4\}) & \rightarrow 11 \\
w(\{V_1, V_4\}) & \rightarrow 6 \\
\end{align*}
\]
Instead: Let's create a recurrence relation for the MWIS.
• Previous Example of Recurrence Relation
  Let \( T(n) \) be \# of \( n \) bit strings with even \# of 1’s.

\[
\begin{array}{c}
\begin{array}{c}
\cdots \quad 0 \\
\end{array}
\end{array}
\]

\[
\begin{array}{c}
\begin{array}{c}
1 \\
\end{array}
\end{array}
\]

Final bit has two options, 0 or 1

\[
T(n) = T(n-1) + \quad \text{if 0}
\]

\[
\text{if 1}
\]

Now to MWIS Problem:

1. Consider options for optimal solution \( S \)

Last vertex has two options

i) \( v_n \notin S \)

ii) \( v_n \in S \)
2. Relate $S$ to solution of smaller problem for each option

Last vertex has two options
i) $v_n \notin S$
ii) $v_n \in S$

i) If $v_n \notin S$, □ is MWIS on the graph □

Why? (Try proof by contradiction.)

ii) $v_n \in S$, □ is MWIS on the graph □

Why? (Try proof by contradiction.)
2. Relate $S$ to solution of smaller problem for each option

For each option:

i) If $v_n \notin S$, $S$ is MWIS on $G_n$.
   
   Pf: $S$ is a valid indep. set for $G_n$, so need to show has max weight.
   Suppose for contradiction.
   $S'$ is an indep. set of $G_{n-1}$ with larger weight than $S$.
   Then $S'$ is also a larger weight indep. set on $G_n$ than $S$, so $S$ is not MWIS of $G_n$, a contradiction.

ii) $v_n \in S$, $S - v_n$ is MWIS on $G_{n-2}$.
   
   Pf: $S - v_n$ is a valid indep. set for $G_{n-2}$, so need to show has max weight.
   Suppose for contradiction.
   $S'$ is an indep. set of $G_{n-2}$ with larger weight than $S - v_n$.
   Then $S' \cup v_n$ will have larger weight than $S$ on $G_n$, a contradiction.