Goals

- Describe the connection between counting and probability
- Describe sample space, events, probability of an event
- Determine the probability of events.

Midterm

- Everything except probability (up to PS 8)
- Post to Canvas Discussion tonight to influence Wed. review
- Same system as first midterm

Q: If 8 people from a basketball team show up to a game, how many ways are there to form a 5 person team?

A) 40  B) 56  C) 60  D) 112

Q:

How many DNA strings of length 5, i.e. strings in \{C, T, G, A\}^5 have exactly 3 C's, and no other repeated letters?
Q: If 8 people from a basketball team show up to a game, how many ways are there to form a 5 person team?

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\[
\frac{8!}{5!3!} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 8 \cdot 7 = 56
\]

How many DNA strings of length 5, i.e. strings in \{C,T,G,A\}^5 have exactly 3 C's, and no other repeated letters?

\[
\text{Task 1}\quad \frac{\text{Choose where to put C's:}}{\binom{5}{2} = \frac{5!}{2!3!} = 10}
\]

\[
\uparrow \quad \text{Choose 2 out of 5 positions to have C's}
\]

\[
\text{and} \quad \frac{\text{Task 2}}{\binom{3}{2}} = \frac{3!}{1!} = 6
\]

\[
\text{Choose 2 of the remaining 3 letters to put in remaining 2 spots (order matters)}
\]

\[
P(3, 2) = \frac{3!}{1!} = 6
\]
Probability

Probability is all about counting.

Ex: What is the probability that the outcome of a die roll is at least 5?

1. Count all outcomes
\[ |\{1, 2, 3, 4, 5, 6\}| = 6 \]

2. Count outcomes where rolled value is \geq 5
\[ |\{5, 6\}| = 2 \]

3. Take ratio:
\[ \frac{2}{6} = \frac{1}{3} \]
Terminology

- Sample space: set of all possible outcomes.
- Event: subset of sample space.

\[ \text{def: If all elements in sample space } S \text{ are equally likely, the probability of an event } E \text{ is} \]
\[ \Pr(E) = \frac{|E|}{|S|} \]

Note: Probability that an event \( E \) does NOT happen

\[ 1 - \Pr(E) \]

\[ \star \text{ Sometimes easier to calculate the probability of an event not happening, and use this rule to find the probability of an event happening.} \star \]
Q: Lottery where a 4-digit number is chosen randomly. What is the size of the sample space?

A) 40  B) 10,000  C) \(10^4\)  

\[\uparrow\]

Use product rule:
\[10 \times 10 \times 10 \times 10\]

Q: Suppose you win some money if you get 3 of 4 numbers matching.

Ex: Lotus: 5 7 0 1
You: 5 7 5 1

What is the probability you get 3 out of 4 correct?

A) \(3/10^4\)  B) \(10/10^4\)  C) \(36/10^4\)  D) \(40/10^4\)

\[\uparrow\]

Sum rule: \(9 + 9 + 9 + 9 = 36\)

or Product rule: \(\binom{4}{3} \times 9 = 36\)
0. What is the probability that you **DONT** win lottery?
(Don't get 3 or 4 digits the same)

\[ \begin{align*} 
\text{A) } 0.9962 & \quad \text{B) } 0.9963 & \quad \text{C) } 0.9964 & \quad \text{D) } 0.9965 \\
\uparrow & \\
\text{36 ways to match 3 #'s} & \quad \text{1 way to match 4 #'s} \\
\text{37 ways to win} & \\
\Pr(\text{NOT win}) = 1 - \frac{37}{10,000} = 0.9963 
\end{align*} \]