Goals: 
- Implement Deduction Strategies
- Create sets using set builder notation

Deductions: Known true statements → new true statements

\[ P \rightarrow Q \]

Premises:
\[ P \]
\[ Q = T \]
\[ = T \]

\[ \therefore \text{You passed a swim test.} \]

Conclusion:
\[ \therefore \text{You passed a swim test.} \]

2 Strategies

1. Truth table. Cross out false rows, see what is left

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>P → Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
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<tr>
<td>T</td>
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</table>
2. Reason it out.

If P is true and P \rightarrow Q is true then Q must be true because otherwise T \rightarrow F = F.

Q:

Andre has a black suit and a tweed suit. He always wears his tweed suit OR he wears sandals. If he wears his tweed suit and purple shirt, he does not wear a bow tie. He never wears his tweed suit unless he also wears a purple shirt OR sandals. If he wears sandals, he also wears a purple shirt. Yesterday, Andre wore a bow tie. What else did he wear?

OR = logical or

W = tweed suit
P = purple shirt
S = sandals
B = bow tie
\[ W = \text{tweed Suit} \]
\[ P = \text{purple shirt} \]
\[ S = \text{sandals} \]
\[ B = \text{bow tie} \]

\[ W \land P = F \]

\[ W \lor S \]
\[ W \land P \rightarrow \neg B \]
\[ W \rightarrow (P \lor S) \]
\[ S \rightarrow P \]
\[ B \]

\[ \neg W \]
\[ +W \lor S \]
\[ S = T \]
\[ +S \rightarrow P \]
\[ P = T \]

\[ P = F \]
\[ +S \rightarrow P \]
\[ S = F \]
\[ +W \lor S \]
\[ W = T \]
\[ W \rightarrow (P \lor S) = F \]

[Cross mark]
W → S
W → A → ¬B
W → (P ∨ S)
S → P
B

WAP is false

Solution

<table>
<thead>
<tr>
<th>W</th>
<th>P</th>
<th>S</th>
<th>B</th>
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Sets

Set is a group of unordered objects (no repeats, order doesn’t matter)

Metaphor: Folder on computer

- Contains files + other folders
- Could be empty
Roster Notation: \( A = \{0, 2, 5\} \) means "\( A \) is the set containing the elements 0, 2, 5."

for sets "element" = "object"

\( \in \): \( 2 \in A \) means 2 is an element of A

\( \notin \): Prof. Watson \( \notin S \) means Prof. Watson is not an element of S.

Sets in Sets: \( T = \{x, y, \{g, h\}, k\} \)

an element of a set can be another set

Q: Is \( g \in T \)? Is \( \{g, h\} \in T \)?

elements of \( T \) are \( x, y, \{g, h\}, k \)

Also \( \{x, y\} \notin T \)
Famous Sets

\[ \emptyset = \text{empty set} = \{ \} \]
\[ \mathbb{N} = \text{set of natural numbers} = \{ 1, 2, 3, \ldots \} \]
\[ \mathbb{Z} = \text{set of integers} = \{ \ldots -3, -2, -1, 0, 1, 2, 3, \ldots \} \]
\[ \mathbb{R} = \text{set of real numbers} \]
\[ \mathbb{Q} = \text{set of rational numbers (fractions) } \]

**NOTE:** In some books, \( \mathbb{N} = \{ 0, 1, 2, 3, \ldots \} \) starts at 0.

Set Builder Notation

\[ \{ f(x) : P(x) \} = \text{the set of } x \text{ where } P(x) \text{ is true, with } f(x) \text{ applied to } \]
\[ \text{function of } x \]
\[ \text{predicate of each element} \]

**Ex:** \[ A = \{ x^2 : x \text{ is even} \} = \{ 0^2, 2^2, 4^2, 6^2, \ldots \} \]
\[ = \{ 0, 4, 16, 36, \ldots \} \]

\[ A = \{ (2x)^2 : x \in \mathbb{Z} \} = \{ (2 \cdot 0)^2, (2 \cdot 1)^2, (2 \cdot 2)^2, \ldots \} \]
\[ = \{ 0, 4, 16, \ldots \} \]

\[ A = \{ x : x \in \mathbb{N} \land \frac{1}{x} \in \mathbb{Z} \} \]