Math Foundations of Computer Science
Inductive Proof Recipe:

• Let $P(n)$ be the predicate ___________. We will prove, using induction on $n$, that $P(n)$ is true for all $n \geq __$

• Base Case: $P(__)$ is true because ___________

• Inductive Case: Let $k \geq __$. Assume, for induction, that $P(k)$ is true. Then ___[a bunch of explanation and math here]__. Thus, $P(k + 1)$ is true.

• Therefore, by induction, $P(n)$ is true for all $n \geq __$.

Prove: $2^n + 1 \leq 3^n$ for all integers $n \geq 1$.
Prove: Sum of first $n$ odd numbers is $n^2$. 
Set-up

Let $P(n)$ be the predicate $2^n + 1 \leq 3^n$. We will prove via induction that $P(n)$ is true for all $n \geq 1$. 
Base Case

- $P(1)$ is true because $2^1 + 1 = 3^1$. 
Inductive Case

Let $k \geq 1$. Assume for induction that $P(k)$ is true. This means $2^k + 1 \leq 3^k$.

Multiplying both sides by 2 and then subtracting 1, we get $2^{k+1} + 1 \leq 2 \times 3^k - 1$.

Now, $2 \times 3^k = 2^k + 3^k - 3^k = 3^{k+1} - 3^k$. Plugging in: $2^{k+1} + 1 \leq 3^{k+1} - 3^k - 1$.

Now since $-3^k - 1 \leq 0$, we finally have $2^{k+1} + 1 \leq 3^{k+1}$.

Thus $P(k + 1)$ is true.
Conclusion

• Therefore, by induction on $n$, $P(n)$ is true for all $n \geq 1$. 