Functions

All this notation is important because when we write about functions we need to have accurate words.

ex: Common affiliation

\[ f \]

You give the function a student name as input, it gives a grade as output.

We write:

- \[ f : S \rightarrow G \]
- means "\( f \) is a function from domain \( S \) to codomain \( G \)"
- \( f(\text{Carol}) = \text{Atwater} \)
- Atwater is "image" of Carol
- Carol is "preimage" of Atwater
3 important properties

**Surjection**  "Onto"

- Surjective
- Not surjective

**Injection**  "One-to-one"

- Injective
- Not injective

**Injective & Surjective** = **Bijective**
1. Let \( f : \{1, 2, 3\} \to \{1, 2, 3\} \). Which of the following is an incorrect representation of \( f \)?

\[
\begin{array}{c|c|c}
\text{A} & \text{B} & \text{C} \\
1 & \rightarrow & 3 \\
2 & \rightarrow & 2 \\
3 & \rightarrow & 1 \\
\end{array}
\]

\[
\begin{array}{c|c|c|c}
\text{D} & \text{E} & \text{F} \\
1 & \rightarrow & 2 \\
2 & \rightarrow & 3 \\
3 & \rightarrow & 1 \\
\end{array}
\]

2. Consider a function \( f : \mathbb{R} \to \mathbb{R} \). If \( y = f(x) \), we can depict \( f \) as a graph. Which of the following graphs (if any) could depict \( f \)?

![Graphs](image)

3. Think of a real world function that is

- surjective
• injective
• bijective
• neither surjective nor injective

4. When you write a function in python or a method in java, what are typical domains and co-domains?

5. A relation \( R \) from the set \( A \) to \( B \) is a subset of \( A \times B \). That is, \( R \subseteq A \times B \). Explain why we can think of every function as a relation. Prove true or prove false that every relation represents a function.

6. English to Math, and just plain English: explain in words (using the new vocabulary you’ve learned) what each of the following means, and then express each using only mathematical notation.

   (a) A function \( f : S \rightarrow G \) is surjective \( \equiv \)
   (b) A function \( f : S \rightarrow G \) is injective \( \equiv \)

7. A function is strictly increasing if \( f(x) < f(y) \) whenever \( x < y \). A function is increasing if \( f(x) \leq f(y) \) whenever \( x < y \).

   (a) Prove that a if \( f : \mathbb{R} \rightarrow \mathbb{R} \) is strictly increasing, then it is injective.
   (b) Prove that there exists an increasing function that is not injective.

8. Let \( f : \mathbb{Z} \rightarrow \mathbb{N} \) be the function \( f(x) = x^2 \). For each of the following, please give an explanation.

   (a) [6 points] Is \( f \) surjective?
   (b) [6 points] Is \( f \) injective?