Goals

- Prove whether relation is equivalence relation
- Use summation notation for time complexity

Announcements

- Reflection: Practical, more examples b/f group work, go over hw, go over picker;
- Wed reminder

Equivalence Class and Equivalence Relation

Given a set \( A \), can divide into disjoint subsets that have common property.

\[ A = \{ -1, 1, 2, 3 \} \]

**Same Sign**

- \(-1\)
- \(1\), \(2\), \(3\)

**Equal to Self**

- \(-1\)
- \(1\), \(2\), \(3\)

**Even vs. Odd**

- \(-1\)
- \(1\), \(3\)

- \(2\)

Every element in subset is equivalent to every other element in subset.
Another way to express this equivalence is through equivalence relation:

\[(a, b) \in R, \text{ \(R\) an equivalence relation,} \iff a, b \text{ in same equivalence class}\]

\[
def: R \subseteq A \times A, R \text{ reflexive, symmetric, transitive}
\]

\[
\forall a \in A, (a, a) \in R
\]

\[
\forall a, b \in A, (a, b) \in R \implies (b, a) \in R
\]

\[
\forall a, b, c \in A, (a, b) \land (b, c) \implies (a, c)
\]
A) Equivalence Relation
B) Not reflexive
C) Not symmetric
D) Not transitive

1. \[ R = \{ (a, b) \in \mathbb{R} \times \mathbb{R} : a - b \in \mathbb{Z} \} \]

A) Equivalence Relation

1. Let \( a \in \mathbb{R} \). Then \( a - a = 0 \), and \( 0 \in \mathbb{Z} \), so Reflexive condition satisfied.

2. Let \( a, b \in \mathbb{R} \). Assume \( (a-b) \in \mathbb{Z} \). Then \( -(a-b) \in \mathbb{Z} \). For the backward direction.

3. \( a-b = x \in \mathbb{Z} \) \( b-c = y \in \mathbb{Z} \)

\[ x+y \in \mathbb{Z} \]
\[ x+y = (a-b) + (b-c) = a - c \Rightarrow a - c \in \mathbb{Z} \] \( \checkmark \) Transitive

2. \( R \subseteq \mathbb{Z} \times \mathbb{Z} \), \( (a, b) \in R \iff a \mid b \)

C) NOT Symmetric

\( a \mid b \iff b \mid a \) 2 divides 4 but 4 does not divide 2
Input: Adj Matrix $A$ for $G = (V, E)$ (undirected, unweighted, no self loops)

Output:
1. $S = 0$
2. for $i = 1$ to $|V|$
3. for $j = 1$ to $i$
4. $S += A(i, j)$
5. return $S$

How many operations?

For loop $\rightarrow$ summation. Write outer to inner:

$$\sum_{i=1}^{1v1} \left( \sum_{j=1}^{1v1} 1 \right) = \sum_{i=1}^{1v1} i$$

$\uparrow$ outer loop $\uparrow$ inner loop $\# \text{ of ops}$

Analyze inner to outer
\[ \sum_{i=1}^{\left| V \right|} i = 1 + 2 + 3 + 4 + \ldots + \left| V \right| - 1 + \left| V \right| \]

How many pairs? \( \frac{\left| V \right|}{2} \)

Total: \( \frac{\left| V \right| \left( \left| V \right| + 1 \right)}{2} = \frac{\left| V \right| \left( \left| V \right| + 1 \right)}{2} = O(\left| V \right|^2) \)

\[ \sum_{i=1}^{n} i = \frac{n (n+1)}{2} \]
\[ \sum_{i=a}^{b} i = \frac{(a-b+1)(a+b)}{2} \]

So from previous, \( \# \text{ ops} = O(\left| V \right|) \)
Input: Adj Matrix A for $G = (V, E)$ (undirected, unweighted, no self loops)

Output:
1. $S = 0$
2. for $i = 1$ to $|V|$: \[ \sum_{j=1}^{i} O(|V|) \sum_{j=1}^{i} O(|V|^2) \]
3. for $j = 1$ to $i$: \[ S + A[i, j] \]
4. \[ return S \]
5. big-O is upper bound