CS200 - Problem Set 4

Due: Monday, Mar. 12 to Canvas before class

Please read the sections of the syllabus on problem sets and honor code before starting this homework.

1. **[6 points]** What can you deduce from the following true statements? Please write the simplest new true statement possible. (Partial credit will be given for more complex statements.)

$$\begin{array}{l} P \rightarrow R \\ Q \rightarrow R \\ P \vee Q \\ \hline \therefore \end{array} \tag{1}$$

2. **[0 points]**
(Challenge problem) You meet two spiders on the road. Everyone knows that a spider either always tells the truth, or always lies. The first spider says, "If we are brothers, then we are both liars." The second spider says, "We are cousins or we are both liars." Are both spiders telling the truth? (Hint, create a truth table for their statements and consider the possible cases of each spider lying or telling the truth, and use deduction to see if there is a contradiction. Also, can brothers be cousins?)

3. Read section 5.3 of Proof by Richard Hammack. Then read the following poorly written proof of the statement: If n is even, then n^2 is even.

   Proof:

   1. Let $n =$ an integer.
   2. Suppose n is even.
   3. Then $n = 2k$.
   4. $n^2 = (2k)^2$, $(2k)^2 = 4k^2$, so $4k^2 = 2(2k^2)$
   5. Since $(2k^2)$ is an integer, I've shown it is even.

   (The sentences in the proof are numbered to make it easier to reference specific lines in your answer.)

   (a) **[1 point per guideline violation found]** Identify sentences that violate Hammack's mathematical writing guidelines and explain why. (A sentence can violate multiple guidelines, and so can be included multiple times.)

(b) [**6 points**] Rewrite the proof so that it follows Hammack's mathematical writing guidelines.

4. *Pigeon Hole Principle* [**11 point each**] The pigeonhole principle is an extremely important tool in computer science (see this StackExchange post for just some of its many diverse applications). It states: If you put $n+1$ pigeons in n cubbies, there must be a cubby with more than one pigeon in it. Create two proofs of this fact, one that uses proof by induction and one that is a contrapositve proof. Each proof should be graded using the standard 11-point scale.

5. [**11 points**] Prove $\forall n \in \mathbb{Z}$, n is even if and only if $5n+3$ is odd. Prove one direction using a direct proof, and one direction using a contrapositive proof.

6. [**11 points**] Prove that you can make any postage greater than or equal to 14 cents using 3-cent stamps and 7-cent stamps. (Hint - combine proof by cases and induction.)

7. [**3 points**] When we use a direct proof to prove $P \rightarrow Q$ is true, we start by assuming P is true. Why do we not also consider the case that P is false?

8. How long did you spend on this homework?