CS200 - Problem Set 2
Due: Monday, Feb. 26 to Canvas before class

Please read the sections of the syllabus on problem sets and honor code before starting this homework.

1. Don’t forget induction! [11 points]

Prove that Algorithm 1 correctly searches an array of integers for a specific integer. Hint: let \( P(n) \) be the predicate: \texttt{Search} works correctly on an input array of size \( n \). Hint: take a look at the previous week’s solution to remind yourself about the general strategy for algorithms.

\textbf{Algorithm 1: Search}(s, A)

\textbf{Input} : Integer \( s \), and an array of integers \( A \)
\textbf{Output}: Returns \( i \) such that \( A[i] = s \), or \(-1\), if \( s \) is not in the array. (The first element of \( A \) is at position 1.)

\begin{enumerate}
\item \( n = \text{length of } A; \)
\item /* Base Case */
\item if \( n == 1 \) then
\item if \( A[1] == s \) then
\item return \( n; \)
\item else
\item return \( -1; \)
\item end
\item /* Recursive case: */
\item else
\item if \( A[n] == s \) then
\item return \( n; \)
\item else
\item return \texttt{Search}(s, A[1 : n - 1]);
\item end
\item end
\end{enumerate}

2. Set Builder to Roster Notation [2 point each]

The following sets are described in set builder notation. Describe each of them in roster notation, instead. The following symbols are used: \( \mathbb{Z} \) denotes the set of integers; \( \mathbb{R} \) denotes the set of real numbers; \( \mathbb{N} \) denotes the set of natural numbers, i.e., \( \mathbb{N} = \{1, 2, \ldots \} \).

(a) \( \{r : r \in \mathbb{R} \text{ and } r = r^2\} \)
(b) \( \{n : n \in \mathbb{N} \text{ and } n > n^2\} \)
(c) \{x : x is a letter in the word *accommodate*\}
(d) \{z^2 : z \in \mathbb{Z} and 6 < z^3 < 160\}.
(e) \{S \subseteq \{2, 4, 6, 8\} : S \cap \{2, 4\} \neq \emptyset \text{ and } |S| \text{ is even}\}

3. *Set builder notation* [3 points each] Write each of the following sets using set-builder notation:

(a) \(A = \{\ldots, 1/8, 1/6, 1/4, 1/2, 2, 4, 6, 8 \ldots\}\)
(b) \(B = \{1, 4, 8, 16, 32, \ldots\}\)
(c) \(A \cap B\)
(d) Express the set of all sets of 2 integers such that the two numbers in the set are non-zero, have opposite signs, and the magnitude of one of them is the square of the magnitude of the other.

4. *Universal Set* [2 points] Let \(U\), the universal set, be the set of even integers from 2 to 12 inclusive, and let \(A = \{4, 6, 7, 9\}, B = \{2, 3, 4, 5, 7\}\). What is \(\overline{A} - B\)?

5. *Set Operations* [2 points each] Simplify each of the following expressions, where \(A\) is an arbitrary set, \(\emptyset\) is the empty set, and \(U\) is the universal set. Hint: each answer to (a)-(h) is one of \(A, U,\) or \(\emptyset\). Just write down the answer: no proof needed, no steps need be shown.

(a) \(A \cap U\)
(b) \(A \cap \emptyset\)
(c) \(A \cup U\)
(d) \(A \cup \emptyset\)
(e) \(A \cup A\)
(f) \(A \cap A\)
(g) \(A \cup \overline{A}\)
(h) \(A \cap \overline{A}\)

6. *Statements* [3 points each] For each of the following sentences, decide whether it is a statement, predicate, or neither, and explain why

(a) Call me Ishmael.
(b) The universe is supported on the back of a giant tortoise.
(c) \(x\) is a multiple of 7.
(d) The next sentence is true.
(e) The preceding sentence is false.
(f) The set \(\mathbb{Z}\) contains an infinite number of elements.
7. *Statements* [2 point each]

This problem has been postponed until next week’s problem set!! If you’ve already done it, that is OK - but please include your solution for next week, too.

Simplify each of the following expressions, where $p$ denotes a statement, and $T$ and $F$ are the Boolean constants *true* and *false*. Hint: each answer is one of $p$, $T$, or $F$. No proof needed, no steps need be shown.

(a) $T \land p$
(b) $F \land p$
(c) $T \lor p$
(d) $F \lor p$
(e) $p \lor p$
(f) $p \land p$
(g) $p \lor \neg p$

8. How long did you spend on this homework?