Please read the sections of the syllabus on problem sets and honor code before starting this homework.

1. *Don’t forget induction!* [11 points]

   Prove that Algorithm 1 correctly searches an array of integers for a specific integer. Hint: let \( P(n) \) be the predicate: \texttt{Search} works correctly on an input array of size \( n \). Hint: take a look at the previous week’s solution to remind your self about the general strategy for algorithms.

   **Algorithm 1: Search\((s, A)\)**

   **Input**: Integer \( s \), and an array of integers \( A \)

   **Output**: Returns \( i \) such that \( A[i] = s \), or \(-1\), if \( s \) is not in the array. (The first element of \( A \) is at position 1.)

   1. \( n = \text{length of } A; \)
   2. /* Base Case */
   3. if \( n == 1 \) then
   4.     if \( A[1] == s \) then
   5.         return \( n \);
   6.     else
   7.         return \(-1\);
   8.     end
   9. /* Recursive case: */
10. else
11.     if \( A[n] == s \) then
12.         return \( n \);
13.     else
14.         return \texttt{Search}\((s, A[1:n-1]);\)
15.     end

2. *Set Builder to Roster Notation* [2 point each]

   The following sets are described in set builder notation. Describe each of them in roster notation, instead. The following symbols are used: \( \mathbb{Z} \) denotes the set of integers; \( \mathbb{R} \) denotes the set of real numbers; \( \mathbb{N} \) denotes the set of natural numbers, i.e., \( \mathbb{N} = \{1, 2, \ldots \} \).

   (a) \( \{r : r \in \mathbb{R} \text{ and } r = r^2\} \)
   (b) \( \{n : n \in \mathbb{N} \text{ and } n > n^2\} \)
(c) \( \{ x : x \text{ is a letter in the word } \text{accommodate} \} \)
(d) \( \{ z^2 : z \in \mathbb{Z} \text{ and } 6 < z^3 < 160 \} \).
(e) \( \{ S \subseteq \{2, 4, 6, 8\} : S \cap \{2, 4\} \neq \emptyset \text{ and } |S| \text{ is even} \} \)

3. \textit{Set builder notation} [3 points each] Write each of the following sets using set-builder notation:

(a) \( A = \{\ldots, 1/8, 1/6, 1/4, 1/2, 2, 4, 6, 8 \ldots \} \)
(b) \( B = \{1, 2, 4, 8, 16, 32, \ldots \} \)
(c) \( A \cap B \)
(d) Express the set of all sets of 2 integers such that the two numbers in the set are non-zero, have opposite signs, and the magnitude of one of them is the square of the magnitude of the other.

4. \textit{Universal Set} [2 points] Let \( U \), the universal set, be the set of even integers from 2 to 12 inclusive, and let \( A = \{4, 6, 7, 9\} \), \( B = \{2, 3, 4, 5, 7\} \). What is \( \overline{A \cap B} \)?

5. \textit{Set Operations} [2 points each] Simplify each of the following expressions, where \( A \) is an arbitrary set, \( \emptyset \) is the empty set, and \( U \) is the universal set. Hint: each answer to (a)-(h) is one of \( A, U, \) or \( \emptyset \). Just write down the answer: no proof needed, no steps need be shown.

(a) \( A \cap U \)
(b) \( A \cap \emptyset \)
(c) \( A \cup U \)
(d) \( A \cup \emptyset \)
(e) \( A \cup A \)
(f) \( A \cap A \)
(g) \( A \cup \overline{A} \)
(h) \( A \cap \overline{A} \)

6. \textit{Statements} [3 points each] For each of the following sentences, decide whether it is a statement, predicate, or neither, and explain why

(a) Call me Ishmael.
(b) The universe is supported on the back of a giant tortoise.
(c) \( x \) is a multiple of 7.
(d) The next sentence is true.
(e) The preceding sentence is false.
(f) The set \( \mathbb{Z} \) contains an infinite number of elements.
7. **Statements** [2 point each]

This problem has been postponed until next week’s problem set!! If you’ve already done it, that is OK - but please include your solution for next week, too.

Simplify each of the following expressions, where $p$ denotes a statement, and $T$ and $F$ are the Boolean constants *true* and *false*. Hint: each answer is one of $p$, $T$, or $F$. No proof needed, no steps need be shown.

(a) $T \land p$
(b) $F \land p$
(c) $T \lor p$
(d) $F \lor p$
(e) $p \lor p$
(f) $p \land p$
(g) $p \lor \neg p$

8. How long did you spend on this homework?