CS200 - Problem Set 1

Due: Monday, Feb. 19. Upload to Canvas before the beginning of class

Please read the sections of the syllabus on problem sets and honor code before starting this homework.

1. *Errors in Inductive Proofs*

   (a) [**2 points**] DMOI 2.5.14. (You only need to do problem 14 in this section.)

   (b) [**6 points**] Explain what is wrong with the following inductive proof that all Middlebury students have the same eye color. I find it easiest to describe the issue by using the "ladder" analogy from class.

   Proof: Let $P(n)$ be the predicate that any set of n Middlebury students have the same eye color. We will prove $P(n)$ is true for all $n \in \mathbb{N}$ for $n \geq 1$.

   Base case: $P(1)$ is true because any one Middlebury student has the same eye color as themselves.

   Inductive case: Let $k \geq 1$. Assume for induction that any set of k Middlebury students have the same eye color. Now let's consider any set of $k+1$ Middlebury students. If we look at the first k of those $k+1$ students, by our inductive assumption they must all have the same eye color. However, if we look at the last k of those $k+1$ students, by our inductive assumption, they must also all have the same eye color. Now the second student must be part of the first set of k and the last set of k, so all $k+1$ students must have the same eye color as this second student. Thus, any set of $k+1$ Middlebury students have the same eye color.

   Therefore, by induction, $P(n)$ is true for all $n \geq 1$.

2. *Inductive Proofs*

   (a) [**11 points**] Prove using induction that for $n \geq 0$, $7^n - 2^n$ is divisible by 5. (An integer m is divisible by an integer r if $m = r \cdot g$, where g is some other integer.)

   (b) [**11 points**] Prove using induction that $2^n > n^2$ whenever n is an integer, and $n \geq 5$.

   (c) [**11 points**] Prove that $1 + 2 + 3 + \cdots + n = n(n+1)/2$ for any integer n such that $n \geq 1$. (So when $n = 1$, we want to evaluate 1, when $n = 2$, we want to evaluate $1 + 2$, when $n = 3$ we want to evaluate $1 + 2 + 3$, etc.)

   (d) [**11 points**] Finish the following proof that Algorithm 1 correctly multiplies an integer $n \geq 0$ and an integer b.

Algorithm 1: `Mult`(n, b)

```
Input  : Non-negative integer n, and integer b
Output: n × b
/* Base Case                                  */
1 if n == 0 then
2 |  return 0;
3 else
  |  // Recursive step
4 |  return b + Mult(n − 1, b);
5 end
```

Proof: Let $P(n)$ be the predicate: `Mult`(n,b) correctly outputs the product of n and b. We will prove using induction that $P(n)$ is true for all $n \geq 0$.

For the base case, let $n = 0$. In this case, we see the `If` statement is true at line 1, and so the algorithm returns 0. This is precisely what we want, since $0 \times b = 0$ for any integer b, so the algorithm is correct and $P(0)$ is true.

For the inductive step...

3. How long did you spend on this homework?