Goals:
- Deduce new truths
- Use set builder notation appropriately

Another logical symbol: \( \equiv \) "is equivalent to" \( \equiv \) some operator as

**Deduction:** assumed true statements \( \rightarrow \) new true statements

- If you graduated, you passed a swim test.
- You graduated

\[ \therefore \text{You passed a swim test.} \]

\[ P \rightarrow Q \]

\[
\begin{array}{c|c|c}
P & Q & P \rightarrow Q \\
T & T & T \\
T & F & F \\
F & T & T \\
F & F & T \\
\end{array}
\]

If \( P \rightarrow Q \) is true and
\( P \) is true,
\( Q \) must be true

2 Strategies

1. Truth table. Cross out false rows, see what is left

Premises
- \( P = T \)
- \( Q = T \) (assume true, although can be false)

 Conclusion
- \( P \rightarrow Q \) is true.
2. Reason it out:

If $P$ is true and $P \rightarrow Q$ is true then $Q$ must be true because otherwise $T \rightarrow F = F$.

Q: Deduce using a truth table or reasoning:

Layla has black pants and pink pants. They always wear pink pants OR they wear sandals. If they wear pink pants and a green shirt, they don’t wear a bow tie. They never wear pink pants unless they also wear a green shirt OR sandals. If they wear sandals, they also wear a green shirt. Yesterday, Layla wore a bow tie. What else did they wear?

OR=$\lor$ (logical or)
Solve using truth table and/or reasoning

$\lor$ = logical or

$P$ = pink pants
$G$ = green shirt
$S$ = sandals
$B$ = bow tie
\[ P = \text{pink pants} \]
\[ G = \text{green shirt} \]
\[ S = \text{sandals} \]
\[ B = \text{bow tie} \]

1. \[ P \lor S \]

2. \[ P \land G \rightarrow \neg B \]

3. \[ P \rightarrow (G \lor S) \]

4. \[ S \rightarrow G \]

5. \[ B \]

\[ \therefore P \land G = F \]

\[ \begin{align*}
\text{or} & \\
P &= F \\
1. & \\
\therefore S \\
4. & \\
\therefore \neg S \\
\therefore G \\
S, G, B, \neg P
\end{align*} \]

\[ \begin{align*}
G &= F \\
4. & \\
\therefore \neg S \\
(\text{S=F}) \\
\therefore \neg S \\
1. & \\
\therefore P \\
P & \\
3. & \\
\therefore G = T \rightarrow \neg X
\end{align*} \]
P \lor S
\neg P \land G \rightarrow \neg B
P \rightarrow (G \lor S)
S \rightarrow G
B

P \land G \text{ is false}

Solution

\begin{array}{|c|c|c|c|}
\hline
P & G & S & B \\
\hline
T & T & T & T \\
T & T & F & F \\
T & F & T & T \\
F & F & F & F \\
F & F & F & F \\
\hline
\end{array}