Sets

- objects
- group of unordered elements
- no repeats

Metaphor: Folder on computer
- Contains files + folders
- Could be empty
- Can't contain same object twice

\[ \text{set} \]
\[ \text{a set can contain other sets as elements} \]
Roster Notation: \( A = \{0, 2, 5\} \) means "\( A \) is the set containing the elements \( 0, 2, 5 \)."

Since order doesn't matter: \( A = \{2, 0, 5\} \) \( \iff \) same set \( A = \{5, 2, 0\} \)

\( \in \): \( 2 \in A \) is a statement. True if \( 2 \) is an element of \( A \).

\( \notin \): "\( \text{dog} \) \( \notin A \) \iff \neg \text{"dog"} \in A \). True if "\( \text{dog} \)" is not an element of \( A \).

Sets in Sets

\[ T = \{x, y, \{g, h\}, k\} \]

an element of a set can be another set

Q: Is \( g \in T? \) Is \( \{g, h\} \in T? \)


\( \{g, h\} \) \( \in \) \( T \):

\( \text{elements of } T \text{ are } x, y, \{g, h\}, k \)

\( \{x, y\} \notin T \)

*Also \( \{x, y\} \notin T \)
Famous Sets

\( \emptyset \) = empty set = \{ \}\n\( \mathbb{N} \) = set of natural numbers = \{1, 2, 3, \ldots \}
\( \mathbb{Z} \) = set of integers = \{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots \}  
\( \mathbb{R} \) = set of real numbers
\( \mathbb{Q} \) = set of rational numbers (fractions)

**NOTE:** In some books, \( \mathbb{N} = \{0, 1, 2, 3, \ldots \} \)
starts at 0.

Set Builder Notation

\[ B = \{ f(x) : P(x) \} \]  
\( f(x) \) = function of \( x \)
\( P(x) \) = predicate of \( x \)

\( \{ f(x) : P(x) \} \) = the set of \( x \) where \( P(x) \) is true, with \( f(x) \) applied to each element

**Ex:** \( A = \{ x^2 : x \text{ is even} \} = \{0^2, 2^2, 4^2, 6^2, \ldots \} \)
\( = \{0, 4, 16, 36, \ldots \} \)

\( A = \{ (2x)^2 : x \in \mathbb{Z} \} = \{(2 \cdot 0)^2, (2 \cdot 1)^2, (2 \cdot 2)^2, \ldots \} \)
\( = \{0, 4, 16, \ldots \} \)

\( A = \{x : x \in \mathbb{N} \land \sqrt[2]{x} \in \mathbb{Z} \} \)