Prove using induction that the program $\text{Sum}(A)$ outputs the sum of an List $A$

Input : List $A$ of integers

Output: Sum of the elements of $A$.

1 $l=\text{length}(A)$;

// Base Case
2 if $l$ equals 1 then
3  return $A[1]$;
4 else
5  // Recursive step
7    // $A[1:l-1]$ is a list containing the first $l-1$ elements of $A$.
6 end

Algorithm 1: $\text{Sum}(A)$

Solution Let $P(n)$ be the predicate that $\text{Sum}(A)$ outputs the sum of the elements of $A$ for any list of length $n$. We will prove $P(n)$ is true for all $n \geq 1$.

Base case: when $n = 1$, the list only has one element, the sum of all of the elements in the list is just the value of that element. When $n = 1$, the base case triggers in line 2 and we return the value of the one element of $A$, which is correct.

Inductive step: Let $k \geq 1$. We assume for induction that $P(k)$ is true. Let’s analyze what happens when the input to $\text{Sum}$ is a list with $k+1$ elements. Since $k \geq 1$, $k+1 \geq 2$, so the algorithm goes to the recursive step in line 5, and returns $\text{sum}(A[1:l-1]) + A[l]$. Since $A[1:l-1]$ is a list with $k$ elements, by inductive assumption, $\text{sum}(A[1:l-1])$ correctly returns the sum of the $k$ elements, which is the sum of the first $k$ elements of $A$. But now the sum of all $k+1$ elements of $A$ is just the sum of the first $k$ elements, plus the final element. This is precisely what line 5 returns, so the outcome is correct.

Therefore, by induction $P(n)$ is true for all $n \geq 1$. 